Factional Conflict and Territorial Rents*

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Abstract

I study a model of conflict between armed factions over economically valuable territory. Factions fight over control of fixed territories within which rents are endogenously generated through the exercise of market power. The amount of market power, and thus the level of conflict, depends on geography, population density, transportation costs, the number of factions, and the level of market consolidation. Consistent with standard intuitions, changes to conditions in the country as a whole that increase market power or market size lead to an increase in rents and an increase in conflict. However, contrary to these same intuitions, changes in local conditions that lead to an increase in the economic rents associated with controlling a territory reduce, rather than increase, conflict. Increasing the number of factions results in more frequent, but less intense, violence. As a consequence, increased factionalization is associated with a decrease in the variance in violence, but has a non-monotone relationship to the expected level of violence.

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1 Introduction

In many settings, armed factions compete for control of territory that can be used to extract local rents.[1] For instance, such conflict characterizes the relationship between rival drug gangs fighting over street corners, Afghan warlords competing for control over transportation routes and checkpoints, Mexican drug trafficking organizations fighting over territory on which they can act as monopsonists relative to local farmers, Colombian rebels fighting over control of territory where they can tax or control the illicit drug trade, and so on. Somewhat more speculatively, one could argue that this sort of conflict over territorial rents typified the wars that led to the rise of the modern state in early modern Europe (Bean, 1973; Tilly, 1992; Besley and Persson, 2009). A recent empirical literature is increasingly interested in the relationship between territorial control, local rents, and violent conflict. (See, for example, Angrist and Kugler (2008); Castillo, Mejia and Restrepo (2013); Mejia and Restrepo (2013); Dell (2014); Dube, García-Ponce and Thom (2014).)

This paper investigates the relationship between factionalization, territorial control, geography, local market power, and conflict. In the model, factions fight over control of spatially differentiated territories that allow for the exercise of market power. After territorial control is established, factions compete over prices for a single good sold at each territory.[2] There are territorial rents because consumers bear transportation costs for getting to more distant locations. Factions fight over access to these endogenously determined rents. Two exogenous factors affect the amount of rents that can be extracted from a given territory: transportation costs and population density near that territory. In addition, the number of factions and pattern of territorial control affect the magnitude of local market power and, thus, the endogenously determined rents associated with taking new territory. Hence, all of these factors affect violent outcomes.

The analysis of these determinants of violence generates two types of insights. First, the analysis directly yields theoretical predictions about the relationship between various observable features of the environment and conflict outcomes. These predictions constitute testable hypotheses. The second type of insight is somewhat more conceptual. The theoretical conflict literature, focusing on providing an account of the underlying causes of conflict, typically assumes that groups compete over a prize of fixed value (see Jackson and

[1]For theoretical models of conflict over economic rents (though typically not local rents), see, for example, Hirshleifer (1991); Grossman (1999); Hafer (2006); Fearon (2008); Besley and Persson (2011); Dal Bó and Dal Bó (2011).

[2]The economic model is in the spirit of Salop (1979), but with fixed locations, as in Alesina and Spolaore's (2005) model of state formation, as appropriate for the application.

Morelli, 2011, for an overview). By contrast, in my model, all of the predicted relationships are driven by the fact that the value controlling a particular territory is determined endogenously by future economic behavior, which, in turn, depends on transportation costs, population density, the number of factions, and the pattern of territorial control. Hence, the model highlights, in one setting, the value of endogenizing the returns to conflict for understanding how conflict plays out once it gets started.

How do transportation costs, population density, and the number of factions affect violent outcomes? To answer this, it is first important to understand an intuition from the model of conflict. Changes in different factions' incremental returns to winning control over a given territory have opposite effects on the distribution of violence. An increase in the incremental return to winning for the faction that values winning the most (call this IR_1) tends to decrease conflict by scaring-off other factions. But an increase in the incremental return to winning for the faction that values winning the second most (call this IR_2) tends to increase conflict by reducing scare-off and increasing investment in violence by increasing the stakes of conflict. Often some change in the environment will increase (or decrease) both IR_1 and IR_2, resulting in competing effects. Because IR_2 affects violence through two mechanisms, while IR_1 affects violence through only one mechanism, the effect of IR_2 typically dominates. (This is made precise below.)

I consider two types of comparative statics regarding transportation costs and population density. First, I consider global comparative statics, asking what happens to the distribution of violence when population density or transportation costs change in the country as a whole. Because a global increase in population density or transportation costs is associated with an increase in the stakes of conflict, but no net change in scare-off, such a change increases expected violence.

The local comparative statics are both more interesting and more relevant for empirical research exploiting local variation. Here I ask how the distribution of violence is affected by local variation in the population density or transportation costs just at the territory under dispute. Two surprising facts emerge from this analysis. First, local transportation costs and local population density push in opposite directions. Second, and more importantly, in both cases, increased rents are associated with decreased violence, exactly the reverse of the global comparative statics.

First, consider local transportation costs. An increase in consumers' costs of getting to a particular territory decreases the marginal costs to raising prices (in terms of foregone demand) for the factions that control surrounding territories. Hence, when local transportation costs at one territory increase, prices at the surrounding territories increase. This spills over into an increase in prices in all territories, which increases rents for all factions. Importantly, the rents increase more slowly for whichever faction ends up with control over the territory with increased transportation costs because of the direct negative effect of increased local transportation costs on demand at that territory. As a consequence, while both factions' rents are increasing in local transportation costs and both factions' incremental returns to winning control over the territory are positive, those incremental returns are decreasing in local transportation costs. For the attacker, the rents are going up less quickly if it takes the territory than if it doesn't and for the defender, the rents are going up less quickly if it holds on to the territory than if it loses it. The two incremental returns change at similar rates and so, as anticipated above, the effect of the smaller incremental return (here, the defender's) dominates. This means that rents are increasing, but expected violence is decreasing, as local transportation costs increase.

Next consider local population density. An increase in local population density at a particular territory increases the marginal costs to raising prices (in terms of foregone demand) for the factions that control surrounding territories. Hence, as local population density goes up, prices at the surrounding territories go down, which spills over into lower prices at all territories. This price decline tends to reduce rents. However, the rents decrease more slowly for whichever faction ends up with control over the territory with greater population density, since there is a direct positive effect on demand for that faction (indeed, if the population density gets large enough, rents for that faction can be increasing in local population density). As a consequence, both factions' incremental return to winning the conflict is increasing in local population density—for the attacker, the rents are decreasing less quickly if it takes the territory than if it doesn't and for the defender, the rents are decreasing less quickly if it holds on to the territory than if loses it. The two incremental returns change at similar rates and so, again, the effect of the smaller incremental return (here, the defender's) dominates. This means that, even when all rents are decreasing, expected violence is increasing as local population density increases.

The contrast between the local and global comparative statics are important for thinking empirically about the political economy of conflict. Typically, empiricists have assumed that territorial conflict will increase whenever some factor increases local rents. But this intuition comes from thinking about changes akin to my global comparative statics. The model here shows that exactly the opposite is true for local changes. And, as the empirical literature becomes increasingly concerned with identification, this is precisely the kind of variation being studied. Further complicating matters, in the case of local population density, a change that might be expected to increase territorial rents—increasing the size of the market—can actually decrease such rents by increasing competition. Hence, the model also emphasizes the potentially subtle relationship between market features, spatial competition, rents, and the returns to conflict.

In addition to the comparative statics above, I also study the effects of changes to the number of factions. Qualitative accounts and conventional wisdom often suggest that increased factionalization causes an increase in violence.[3] The analysis here suggests matters are more subtle.

Factional consolidation (i.e., a move to fewer factions that each control more territory) is associated with three changes that affect the distribution of violence. First, factional consolidation is associated with greater global market power and, thus, increased incremental returns and incentives for violence. Second, because the economic model has increasing returns to scale, as the factions consolidate, the attacker's incremental returns increase more quickly than do defender's, which affects the amount of scare-off. Third, increased consolidation is associated with the emergence of "safe territories"—territories that are insulated from attack by virtue of being surrounded entirely by other territories controlled by the same faction—which decreases opportunities for conflict. (This is consistent with Papachristos, Hureau and Braga's (2013) finding that gangs are more likely to fight when their territories are adjacent.) These various forces imply that as the number of factions increases (i.e., consolidation decreases), conflict becomes more frequent, but when it occurs the violence is less intense. As a result, the variance in the amount of violence is decreasing in the number of factions, while the net effect of factionalization on expected violence is non-monotone. Finally, the increased frequency of conflict associated with increased factionalization leads to a decrease in the stability of the configuration of territorial control—highly factionalized environments tend to consolidate through territorial conquest.

The remainder of the paper is organized as follows. Section 2 specifies the formal model. Section 3 provides some formal results and intuitions about the distribution of conflict for arbitrary incremental returns to winning the conflict. Section 4 characterizes the rents associated with the economic equilibrium for the case of two factions. Sections 5 and 6 develop global and local comparative statics (respectively), using the results in Sections 3 and 4 to pin down the relevant incremental returns and, thus, equilibrium behavior in the conflict stage. Section 7 does likewise for the effects of factionalization, which requires extending beyond the two faction case. Section 8 concludes.

[3] See, for example, Beittel (2013) or Jeremy Garner. Gang factions lead to spike in city violence. *The Chicago Tribune*, October 3, 2012. Available: http://articles.chicagotribune.com/2012-10-03/news/ct-met-street-gang-bloodshed-20121003_1_gang-violence-gangster-disciples-black-p-stones.

2 The Model

There are six fixed *territories*, labeled $A - F$, located at equal intervals on the perimeter of a circle.[4] Two territories are *contiguous* if they are located next to each other. Let $\leftrightarrows$ indicate that contiguous relationship. The territories are arrayed in alphabetical order:

$$A \leftrightarrows B \leftrightarrows C \leftrightarrows D \leftrightarrows E \leftrightarrows F \leftrightarrows A$$

There is a population of mass N located uniformly on the perimeter of the circle.

The game is played as follows.

(i) At the beginning of the game, there is some configuration of factional control of the territories described by a partition of $\{A, B, C, D, E, F\}$.

(ii) Nature chooses one territory to become *vulnerable*. Any faction that controls either the vulnerable territory or a territory contiguous with it may fight for control of the vulnerable territory. At the end of the conflict either the territory is still controlled by its original owner or has changed hands.

(iii) After the fighting is over, factions set prices for the single good traded in the economy. At each territory, j, it controls, a faction can set a different price, $p_j \in [0, 1]$.

(iv) Each population members decides whether and from whom to buy the good, given the prices and distances.

(v) The game ends.

Conflict is modeled as an all-pay auction. A faction i that is participating in the conflict chooses a level of investment in fighting, $a_i \in \mathbb{R}_+$. Call the ex ante holder of a vulnerable territory the *defender* and all factions with contiguous territories *attackers*. If one of the factions involved in fighting invests strictly more than any other faction, it wins the territory. If the defender is involved in a tie, she wins. If two attackers are involved in a tie, they win with equal probability.[5]

Each population member gets a benefit of 1 from consuming the good. Population members bear linear transportation costs, t. Hence, if a population member buys the good for price p from a territory at distance x from her location, her payoff is:

$$1 - p - tx.$$

[4]Six factions is the smallest number of factions needed for the comparisons in Section 7.

[5]Since ties will never occur in equilibrium, the tie breaking rule is irrelevant.

If she doesn't buy the good, her payoff is zero. To insure interior solutions and that the full market is served, I assume that transportation costs are not too large: $0 < t < 1$.

The factions bear costs for investing in conflict and make profits from selling the good. If a faction makes revenues r and invests a in conflict, its payoff is

$$r - a.$$

The solution concept is subgame perfect Nash equilibrium.

3 Conflict for General Incremental Returns

Before turning to an analysis of the conflict game, it is worth noting an important underlying assumption. In the economic game, the total amount of rents is increasing in market concentration. As such, if the factions could commit to an ex ante arrangement, they would like to behave as a monopolist and then split the rents. The same is true of any particular dispute over a territory. The attacker and defender factions would always be best off agreeing not to fight, allowing the territory to be controlled by the largest of these factions, and then sharing the rents. Hence, the model implicitly assumes some sort of commitment problem among the factions that prevents such ex ante agreements (Fearon, 1995; Powell, 2004). This assumption seems natural for the application.

In the conflict game, a faction deciding how much to invest in fighting for control over the vulnerable territory compares its expected payoff in the economic equilibrium should it win vs. lose the fight. Label a specific configuration of territorial control and vulnerability ξ. Call the difference in faction j's expected equilibrium payoff should it win vs. lose its *incremental return to winning* at ξ, IR_j^{ξ}.

In the model, at most three factions can be involved in conflict. As we will see later, in the configurations of interest, it turns out that, even when three factions can fight, the defender's incremental return is strictly lower than the attackers'. As such, the following results from the literature on all pay auctions are key for the analysis:

Theorem 3.1 *(Hillman and Riley, 1989; Baye, Kovenock and De Vries, 1996) In an all pay auction with linear costs, let IR_i be player i's expected incremental return from winning the auction instead of losing the auction. If either there are two players with $\mathrm{IR}_1 \geq \mathrm{IR}_2$ or there are three players with $\mathrm{IR}_1 \geq \mathrm{IR}_2 > \mathrm{IR}_3$, then there is a unique equilibrium. In it, Player 1 bids the realization of a random variable drawn from the uniform distribution on $[0, \mathrm{IR}_2]$ and Players 2 bids 0 with probability $\frac{\mathrm{IR}_1 - \mathrm{IR}_2}{\mathrm{IR}_1}$ and with the complementary probability* *bids the realizations of an independent random variable drawn from the uniform distribution on* $[0, \mathrm{IR}_2]$*. Player 3 (if she exist) bids zero.*

It is worth noting that there is a subtlety associated with calculating the incremental returns in this model that does not exist in the standard auction setting. In an all-pay auction, a player's incremental return to winning is exogenous to the bids of the other players—it is simply the value of the asset. Here, this need not be the case. Payoffs in the economic equilibrium are sensitive to the configuration of territorial control. Hence, in a conflict potentially involving three factions, a faction's expected payoff should it lose depends on its beliefs about the likelihood of each of the other factions winning, which depends on those factions' strategies. Of course, this dependence is true for the other factions as well. Thus, in this conflict setting, the expected incremental return to winning, which determines equilibrium investment in conflict, need not follow from the economic equilibrium alone—it is endogenous to the investment choices in the conflict game. This fact will have to be attended to when providing a complete characterization of equilibrium. But I can abstract from it here in order to build intuitions for the case of conflicts involving only two active factions, which turns out to be the case of interest.

Here, I focus on what conflict looks like given an arbitrary configuration of territorial control and vulnerability, ξ. Later, I will have to account for the fact that Nature randomly chooses one territory to make vulnerable and that conflict over different territories may be different.

Suppose either two or three factions—$1, 2$ and 3—can fight over the vulnerable territory. Moreover, assume $\mathrm{IR}_1^\xi \geq \mathrm{IR}_2^\xi (> \mathrm{IR}_3^\xi)$, so faction 1 values winning the fight at least as much as does faction 2, and both factions value winning more than faction 3.

Theorem 3.1 indicates that only factions 1 and 2 will put positive probability on being active in the fight (i.e., choosing positive investment). Faction 1 will invest the realization of a uniform distribution on $[0, \mathrm{IR}_2^\xi]$. With probability $\mathrm{IR}_2^\xi / \mathrm{IR}_1^\xi$, faction 2 will also invest the realization of a uniform distribution on $[0, \mathrm{IR}_2^\xi]$ and with the complementary probability, faction 2 will remain inactive, ceding the territory to faction 1.

Violence only occurs if at least two factions make a positive investment. Importantly, when $\mathrm{IR}_2^\xi < \mathrm{IR}_1^\xi$, faction 2 cedes with positive probability. Hence, a conflict does not always result in violence.[6] Conditional on at least two factions being active in conflict, the amount

[6]Here the application of all pay auctions to conflict is importantly different from its application to, say, competition for political influence, where, in equilibrium, actual donations are made by the player that values winning the most, regardless of whether any other players make donations (Becker, 1983; Baye, Kovenock and De Vries, 1996; Krishna and Morgan, 1997).

of violence, f, is the sum of the investments:

$$f = \begin{cases} a_1 + a_2 & \text{if } \min\{a_1, a_2\} > 0 \\ 0 & \text{else.} \end{cases}$$

From an ex ante perspective, the amount of violence is a random variable. With probability $1 - \frac{\mathrm{IR}_2^\xi}{\mathrm{IR}_1^\xi}$, f takes the value 0. And with probability $\mathrm{IR}_2^\xi/\mathrm{IR}_1^\xi$, f is the sum of two uniform random variables on $[0, \mathrm{IR}_2^\xi]$. This implies that, with probability $\mathrm{IR}_2^\xi/\mathrm{IR}_1^\xi$, f has a symmetric triangular distribution on $[0, 2\mathrm{IR}_2^\xi]$. Hence, f has a CDF given by

$$\Phi^\xi(f) = \begin{cases} 1 - \frac{\mathrm{IR}_2^\xi}{\mathrm{IR}_1^\xi} + \frac{\mathrm{IR}_2^\xi}{\mathrm{IR}_1^\xi}\left(\frac{f^2}{2(\mathrm{IR}_2^\xi)^2}\right) & \text{if } f \in [0, \mathrm{IR}_2^\xi] \\ \\ 1 - \frac{\mathrm{IR}_2^\xi}{\mathrm{IR}_1^\xi} + \frac{\mathrm{IR}_2^\xi}{\mathrm{IR}_1^\xi}\left(1 - \frac{(2\mathrm{IR}_2^\xi - f)^2}{2(\mathrm{IR}_2^\xi)^2}\right) & \text{if } f \in [\mathrm{IR}_2^\xi, 2\mathrm{IR}_2^\xi]. \end{cases} \tag{1}$$

From this fact, it is straightforward to calculate the expected level of violence and its comparative statics.

Proposition 3.1 *Given a configuration of vulnerability and territorial control, ξ, the expected level of violence is*

$$E[f|\xi] = \int_0^{2\mathrm{IR}_2^\xi} f \, d\Phi^\xi(f) = \frac{(\mathrm{IR}_2^\xi)^2}{\mathrm{IR}_1^\xi},$$

which is increasing in IR_2^ξ and decreasing in IR_1^ξ.

Proof. Follows from the analysis in the text. ■

Let's unpack the intuition behind the relationships recorded in Proposition 3.1.

A change to IR_1^ξ has only one effect on equilibrium expected violence. As faction 1's incremental return to winning increases, faction 1 becomes more willing to invest in conflict. Were it to do so, this would make the second faction unwilling to fight at all, since it would be so likely to lose. But if the second faction does not fight at all, then the first faction has no incentive to exert any effort. So to maintain equilibrium, as IR_1^ξ increases, faction 1's increased willingness to invest scares-off faction 2 from staying in the fight. Faction 2 cedes more often, which then establishes equilibrium by decreasing faction 1's incentive to invest in conflict. Thus, this *scare-off* effect tends to reduce the expected amount of violence by increasing the probability that the territory is ceded.

An increase in IR_2^ξ has two effects, both of which tend to increase violence. First, as faction 2's incremental return to winning increases, faction 2 becomes less willing to cede the territory. This *anti-scare-off* effect increases expected violence by increasing the probability that both factions are active. Second, as faction 2's incremental return increases, faction 2 becomes more willing to invest. This *stakes* effect increases both factions' expected investment and, thus, increases expected violence.

Often, in the analysis, some factor will simultaneously increase both IR_1^ξ and IR_2^ξ. Such a change can increase or decrease expected violence, depending on the relative size of the effects on the two incremental returns. Note, however, that IR_2^ξ increases expected violence through two mechanisms—anti-scare-off and stakes—while IR_1^ξ decreases expected violence through only one mechanism—scare-off. Hence, if some factor were to change both incremental returns by similar amounts, the effect on IR_2^ξ would dominate. Indeed, in order for the effect on IR_1^ξ to dominate, it must be more than twice as big.

To see this, suppose that both incremental returns are strictly increasing, differentiable functions of some parameter θ. Then expected violence is decreasing in θ if and only if:

$$\frac{\partial \mathrm{IR}_2^\xi(\theta)/\partial\theta}{\partial \mathrm{IR}_1^\xi(\theta)/\partial\theta} < \frac{\mathrm{IR}_2^\xi(\theta)}{2\mathrm{IR}_1^\xi(\theta)} < \frac{1}{2}. \tag{2}$$

4 Economic Equilibrium

The results in Section 3 show how changes to the incremental returns to winning affect the distribution of violence. I calculate these incremental returns by computing the rents captured by each faction in the economic equilibrium that follows conflict.

Consider two contiguous territories, i and j, charging price p_i and p_j. A population member located between i and j at distance x from i who is choosing between buying from one of these two locations or not purchasing the good, will buy from faction i if:

$$p_i + tx \leq p_j + t\left(\frac{1}{6} - x\right)$$

and

$$1 - p_i - tx \geq 0.$$

The population member who is indifferent between buying from i and j is located at distance

$x_{i,j}^*$ from i, given by:

$$p_i + tx_{i,j}^* \leq p_j + t\left(\frac{1}{6} - x_{i,j}^*\right) \Rightarrow x_{i,j}^* = \frac{1}{12} + \frac{p_j - p_i}{2t}.$$

This population member prefers to purchase rather than exit the market as long as:

$$1 - p_i - tx_{i,j}^* \geq 0.$$

Hence, as long as $p_i \leq 2 - p_j - \frac{t}{6}$, the faction controlling i faces demand from population members located between i and j given by:[7]

$$D_i(p_i, p_j) = \begin{cases} \frac{N}{6} & \text{if } p_i < p_j - \frac{t}{6} \\ N\left(\frac{1}{12} + \frac{p_j - p_i}{2t}\right) & \text{if } p_i \in \left[p_j - \frac{t}{6}, p_j + \frac{t}{6}\right] \\ 0 & \text{if } p_i > p_j + \frac{t}{6}. \end{cases} \tag{3}$$

It will be useful to adopt the notation that territory $i + 1$ is the territory one letter higher in the alphabet than i, except in the case of F, where $F + 1 = A$. Now consider three contiguous territories: $i - 1, i$, and $i + 1$. Assuming all consumers who purchase the good do so from one of the two territories they live closest to (which is true in equilibrium), the rents captured by the faction that owns territory i, given a vector of prices (p_{i-1}, p_i, p_{i+1}), are:

$$p_i \left[D_i(p_i, p_{i-1}) + D_i(p_i, p_{i+1})\right].$$

It is straightforward that for any configuration of territorial control, an equilibrium of the economic game is characterized by six equations (given by the six first-order conditions) and six unknowns. In Appendix A, I characterize the economic equilibrium for all relevant configurations. To fix ideas, consider two cases: two symmetric factions (ABC and DEF) and two asymmetric factions ($ABCD$ and EF).

First suppose there are two symmetric factions, one controlling territories A, B, C and the other controlling territories D, E, F. (I will notate this 3, 3, since there are two factions, each controlling 3 territories.) If demand is characterized by Equation 3 at some vector of prices, the first faction's profits are:

$$\sum_{i=A}^{C} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right],$$

[7] As shown in the appendix, in equilibrium it is always the case that $p_i \leq 2 - p_j - \frac{t}{6}$.

and the second faction's profits are:

$$\sum_{i=D}^{F} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1}) \right].$$

From the first-order conditions, equilibrium prices are given by

$$p_A^* = p_C^* = p_D^* = p_F^* = \frac{t}{2}$$

$$p_B^* = p_E^* = \frac{7t}{12}.$$

Notice two facts. First, prices are higher in interior regions (B and E), reflecting the increased market power associated with safe territory. Second, for all $i, j \in \{A, B, C, D, E, F\}$, we have $p_i \leq 2 - p_j - \frac{t}{6}$, so demand is in fact described by Equation 3. Moreover, it is straightforward that each consumer purchases from one of the two territories to which she is closest.

Equilibrium rents for each faction are:

$$v^{3,3} = \frac{37Nt}{144}.$$

Now suppose there are two factions, one controlling territories A, B, C, D and the other controlling territories E, F. (I notate this $4, 2$.) If demand is characterized by Equation 3 at some vector of prices, the large faction's profits are:

$$\sum_{i=A}^{D} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1}) \right],$$

and the small faction's profits are:

$$\sum_{i=E}^{F} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1}) \right],$$

From the first-order conditions, equilibrium prices are:

$$p_A^* = p_D^* = \frac{5t}{9} \qquad p_B^* = p_C^* = \frac{13t}{18} \qquad p_E^* = p_F^* = \frac{4t}{9}.$$

Note that for all $i, j \in \{A, B, C, D, E, F\}$, we have $p_i \leq 2 - p_j - \frac{t}{6}$, so demand is in fact described by Equation 3. The large and small factions' equilibrium rents, respectively, are:

$$v^{\mathbf{4},2} = \frac{109Nt}{324} \qquad v^{4,\mathbf{2}} = \frac{16Nt}{81}.$$

Again, a couple points are worth emphasizing. First, prices again are higher at interior locations (B and C). Moreover, at border locations, the large faction charges higher prices than the small faction, reflecting its greater market power. Third, because consolidation leads to higher prices, the total amount of rents generated are higher with two unequal factions than with two equal factions. That is,

$$v^{\mathbf{4},2} + v^{4,\mathbf{2}} = \frac{173Nt}{324} > \frac{74Nt}{144} = v^{3,3} + v^{3,3}. \tag{4}$$

5 Global Comparative Statics

In this section I explore how violent outcomes change as population density or transportation costs change for the country as a whole. In the next section I consider local comparative statics—i.e., changes to population density and transportation costs near one particular territory. For the purpose of these comparative statics, I focus on a situation in which there are two symmetric factions—ABC and DEF. (The analysis in Section 7 shows that these comparative statics are not special to this case.)

There are two cases to consider. In the first, the vulnerable territory is one of the border regions (A, C, D, or F). In the second, the vulnerable territory is an interior region (B or E). In the latter case, there is no conflict. Now consider the former case.

For both faction, winning the conflict is valuable. But, as we saw in Condition 4, there are more total rents in the event that the attacker wins. This implies that the incremental return to the attacker must be larger than the incremental return to the defender:

$$v^{\mathbf{4},2} - v^{3,3} > v^{3,3} - v^{4,\mathbf{2}} \iff v^{\mathbf{4},2} + v^{4,\mathbf{2}} > v^{3,3} + v^{3,3}.$$

We can calculate these incremental returns directly The attacker's incremental return is

$$\text{IR}^{3,3}_{\text{att}} = v^{\mathbf{4},2} - v^{3,3} = \frac{103Nt}{1296},$$

while the defender's incremental return is

$$\text{IR}^{3,3}_{\text{def}} = v^{3,3} - v^{4,\mathbf{2}} = \frac{77Nt}{1296}.$$

Given this, conditional on a border territory being vulnerable, the equilibrium outcome follows exactly from the analysis in Section 3, with the attacker having the higher incremental return. This implies that the ex ante distribution over outcomes is a mixture of an atom on zero with probability mass 1/3 and the distribution described in Equation 1 with probability 2/3.

Proposition 5.1 *When the initial configuration involves two factions each of which controls three contiguous territories, if the vulnerable region is interior, there is no conflict. If the vulnerable region is a border region, then equilibrium play at the conflict stage is as follows:*

- *With probability $\frac{77}{103}$ the defender faction's investment is drawn from a uniform distribution on $\left[0, \frac{77Nt}{1296}\right]$ and with complementary probability the defender faction invests zero.*
- *The attacker faction's investment is drawn from a uniform distribution on $\left[0, \frac{77Nt}{1296}\right]$.*

Consequently, the ex ante distribution of violence is given by the following CDF:

$$\Phi^{3,3}(f) = \begin{cases} \frac{1}{3} + \frac{2}{3} \times \frac{26}{103} + \frac{2}{3} \times \frac{77}{103}\left(\frac{1296^2 f^2}{77 \times 154 N^2 t^2}\right) & \text{if } f \in \left[0, \frac{77Nt}{1296}\right] \\ \frac{1}{3} + \frac{2}{3} \times \frac{26}{103} + \frac{2}{3} \times \frac{77}{103}\left(1 - \frac{1296^2\left(\frac{154Nt}{1296} - f\right)^2}{77 \times 154 N^2 t^2}\right) & \text{if } f \in \left[\frac{77Nt}{1296}, \frac{154Nt}{1296}\right]. \end{cases}$$

Proof. Follows from the argument in the text. ■

The global comparative statics are now straightforward. Because both factions' incremental returns are linearly increasing in both the transportation costs and population density, a change to either of those parameters has no effect on scare-off, which is determined by the ratio of the two incremental returns. Thus, a change to global transportation costs or population density affects violence only through the stakes effect. The value of a territory is increasing in both. Hence, the larger are t or N, the larger is $\text{IR}^{3,3}_{\text{def}}$.

These facts have several immediate implications. First, all else equal, an increase in global transportation costs or global population density is associated with a higher level of expected violence. This is consistent with the standard intuition, discussed in the introduction, that an increase in rents leads to an increase in violence (Grossman, 1999). Second, all else equal, an increase in global transportation costs or global population density is associated with an increase in the variance of violence because it has no effect on the probability of no conflict, but increases the upper bound of the support of the distribution of violence. Third, all else equal, an increase in global transportation costs or global population density has no effect on the stability of a factional configuration, since it has no effect on scare-off and, conditional on conflict occurring, the two sides win with equal probability.

Proposition 5.2 *When the initial configuration involves two factions, each of which controls three contiguous territories, the expected level of violence and the variance of violence are increasing in both N and t. The stability of the configuration is unaffected by N or t.*

Proof. See Appendix B ■

6 Local Comparative Statics

Now I turn to comparative statics when the change applies only locally to the territory under dispute. To do so, I again consider a configuration with two equally sized factions. I ask what happens to violence when there are changes to:

(i) transportation costs associated with getting to the territory under dispute, and

(ii) the population density surrounding the territory under dispute.

6.1 Local Transportation Costs

Suppose the costs to a population member of getting to the vulnerable territory increase from t to τt for some $\tau \in [1, 2]$. Since changes to local transportation costs in safe territories do not contribute to violence, in the text I focus on the case where the vulnerable territory is a border territory. To fix ideas, suppose it is territory F. (Of course, all border territories produce the same distribution of violence, so this is without loss.) To compute the incremental returns to winning the fight over territory F, we must calculate the equilibrium rents from the economic game (as a function of τ) in two scenarios: ABC, DEF and $ABCF, DE$.

For a given vector of prices, demand is the same as in Equation 3 in territories B, C, and D but it may be changed in A, E, and F. Fix a vector of prices. As long as $p_F \leq \frac{\tau+1}{\tau} - \frac{p_A}{\tau} - \frac{t}{6\tau}$, for $j \in \{A, E\}$, demand at territory F from the part of the population between F and j is:

$$D_F(p_F, p_j) = \begin{cases} \frac{N}{6} & \text{if } p_F \leq p_j - \frac{\tau t}{6} \\ N\left(\frac{1}{6(\tau+1)} + \frac{p_j - p_F}{t(\tau+1)}\right) & \text{if } p_F \in \left(p_j - \frac{\tau t}{6}, p_j + \frac{\tau t}{6}\right) \\ 0 & \text{if } p_F \geq p_j + \frac{\tau t}{6}. \end{cases} \quad (5)$$

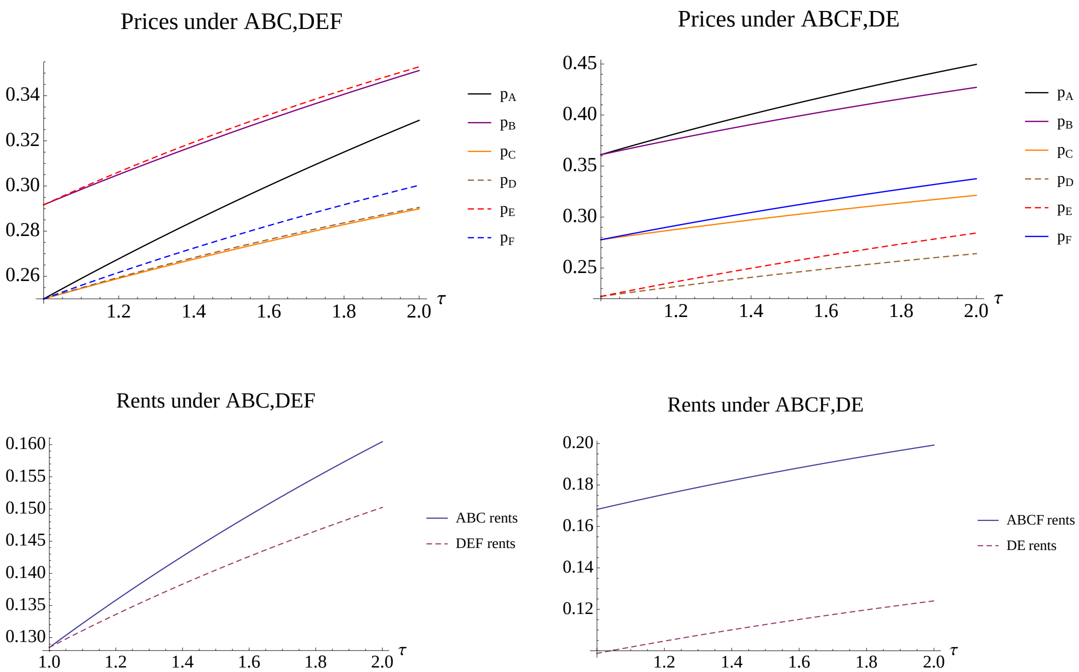

Figure 6.1: When conflict is over territory F, prices and rents are increasing in local transportation costs (τ) at F for both the ABC, DEF and $ABCF, DE$ configurations. The figures are drawn for the case of $t = 1/2$.

For territory $j \in \{A, E\}$, demand from the population between j and F is the complement.

In Appendix B I characterize the economic equilibrium for both configurations of territorial control.

The key facts are illustrated in Figure 6.1. An increase in local transportation costs has two effects. First, there is a direct effect that tends to reduce the rents of the faction that controls F: for a fixed vector of prices, when local transportation costs at F go up, demand at F goes down. Second, there is an indirect effect: when local transportation costs at F go up, the marginal cost (in terms of lost demand) associated with a price increase at A or E goes down. Consequently, prices at A and E go up. Since the economic game has complementarities, this results in prices increasing at all territories, as illustrated in the upper two panels of Figure 6.1, which show that in both configurations, prices are increasing in τ. This indirect effect tends to increase the rents for both factions in both configurations. Moreover, as illustrated in the two lower panels of Figure 6.1 (and formalized in Proposition 6.1), this indirect effect on prices dominates—on net, both factions' rents are increasing in the local transportation costs at F.

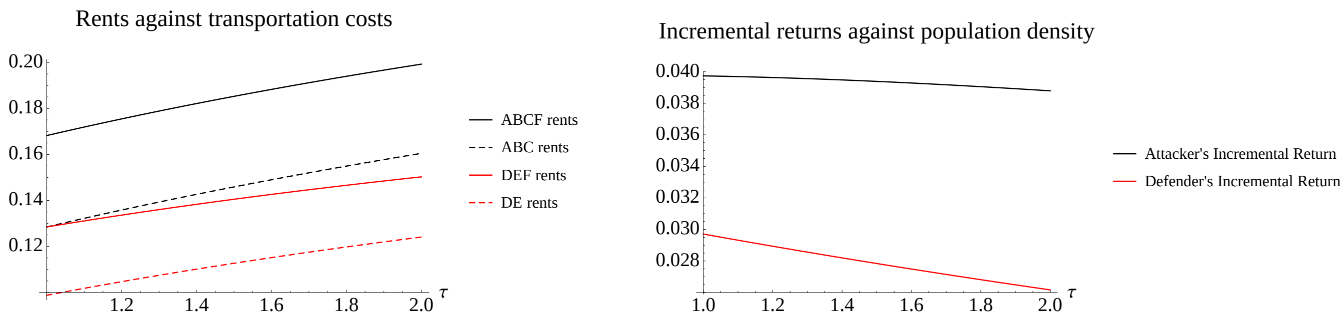

Figure 6.2: Rents are increasing more slowly in transportation costs for the faction that controls F. Hence, incremental returns are decreasing in local transportation costs. The figures are drawn for the case of $t = 1/2$.

The fact that rents are increasing in local transportation costs does not tell us what happens to expected violence. For that, we need to understand the incremental return to winning the territory. And that incremental return is the difference between the rents associated with each of the configurations above. Since rents are increasing for both factions in both configurations, the critical question is about the rate of change.

For a given value of τ, the incremental return to winning for the attacker is:

$$\mathrm{IR}_{\mathrm{att}}^{\mathrm{trans}}(\tau) = v^{\mathbf{ABCF},DE}(\tau) - v^{\mathbf{ABC},DEF}(\tau)$$

and the incremental return to the defender is

$$\mathrm{IR}_{\mathrm{def}}^{\mathrm{trans}}(\tau) = v^{ABC,\mathbf{DEF}}(\tau) - v^{ABCF,\mathbf{DE}}(\tau).$$

These incremental returns reveal a key intuition. Both factions want to win the conflict (i.e., the incremental returns are positive)—controlling territory F is valuable. But, because of the direct effect of local transportation costs on demand at F, controlling territory F decreases the rate at which a faction's rents increase in the local transportation costs. That is, $v^{\mathbf{ABC},DEF}(\tau)$ is increasing faster in τ than is $v^{\mathbf{ABCF},DE}(\tau)$ and $v^{ABCF,\mathbf{DE}}(\tau)$ is increasing faster in τ than is $v^{ABC,\mathbf{DEF}}(\tau)$. Hence, for both factions, although overall rents are increasing in τ, the incremental return to winning the conflict is decreasing in τ. This fact is illustrated in Figure 6.2 (and formalized in Proposition 6.1), where the left-hand cell shows the two component's of each faction's incremental return and the right-hand cell shows the incremental returns themselves (which is the difference between the two components).

The attacker's incremental return is larger than the defender's and both faction's incremental returns are decreasing in τ. As highlighted in Section 3, expected violence is increasing in the defender's incremental return and decreasing in the attacker's incremental return. So how does a change in local transportation costs affect the expected amount of violence? As shown in Equation 2, the effect of the defender's incremental return dominates the effect on the attacker's incremental return, unless the attacker's incremental return changes a lot more than the defender's. This is clearly not the case here. Hence, the effect of an increase in local transportation costs is to increase rents, but decrease expected violence.

Proposition 6.1 *Suppose there are two factions, each of which controls three contiguous territories. Moreover, suppose the transportation costs associated with the vulnerable territory are τt for some $\tau \in [1, 2]$. The following are all true:*

(i) Regardless of what happens at the conflict stage, rents at the economic stage are increasing in τ for both factions.

(ii) When the vulnerable territory is a border territory, the incremental returns to winning the conflict are decreasing in τ for both factions.

(iii) The expected level of violence is decreasing in τ.

Proof. See Appendix B.2. ■

This result is surprising in light of the standard intuitions discussed in Section 5, which motivate much of the empirical literature. That literature hypothesizes that increased rents lead to increased violence. But the results here show this need not be the case. Indeed, with respect to local transportation costs, exactly the opposite holds—when rents increase, incremental returns and expected violence decrease.

6.2 Local Population Density

Now consider a situation in which the population in the 1/6th of the circle surrounding the vulnerable territory is of size $\eta N/6$, for some $\eta \in [1, 2]$. (At $\eta = 1$, this is the baseline model.) Since changes to local population density in safe territories do not contribute to violence, in the text, I focus on the case where the vulnerable territory is a border territory. Again, to fix ideas, suppose it is territory F.

For a fixed vector of prices, demand is the same as in Equation 3 in territories B, C, and D but may be changed in A, E, and F. In particular, assuming $p_A \leq 2 - p_j - \frac{t}{6}$, then at territory F, for $j \in \{A, E\}$, demand is represented by:

$$D_F(p_F, p_j) = \begin{cases} \frac{(1+\eta)N}{6} & \text{if } p_F \le p_j - \frac{t}{6} \\ \frac{\eta N}{12} + N\left(\frac{p_j - p_F}{2t}\right) & \text{if } p_F \in \left(p_j - \frac{t}{6}, p_j\right) \\ \eta N\left(\frac{1}{12} + \frac{p_j - p_F}{2t}\right) & \text{if } p_F \in \left(p_j, p_j + \frac{t}{6}\right) \\ 0 & \text{if } p_F \ge p_j + \frac{t}{6}. \end{cases} \tag{6}$$

For territory $j \in \{A, E\}$, demand from the part of the population between j and F is the complement.

In Appendix B I characterize the economic equilibrium.

The key facts are illustrated in Figure 6.3. An increase in local population density has two effects. First, there is a direct effect that tends to increase the rents of the faction that controls F: for a fixed vector of prices, demand at F goes up. Second, there is an indirect effect: when local population density around F goes up, the marginal cost (in terms of lost demand) associated with a price increase at A or E goes up (if they are competing for that portion of the population). Consequently, pries at A and E go down. Since the economic game has complementarities, this results in price decreases at all territories, as illustrated in the upper two panels of Figure 6.3, which show that, in both configurations, prices are decreasing in η. This indirect effect tends to decrease the rents for both factions in both configurations.

The two lower panels of Figure 6.3 illustrate these effects. Since the faction that does not control territory F has only the indirect effect, that faction's rents are decreasing in local population density. However, for the faction that does control territory F there are competing effects and, as a result, rents are non-monotone in local population density. (These facts are recorded in Proposition 6.2.)

The comparative statics on rents do not tell us what happens to expected violence. For that, we need to understand the incremental returns to winning the territory. The incremental returns are the difference between the rents associated with each of the configurations above.

For a given value of η, the incremental return to winning for the attacker is:

$$\text{IR}_{\text{att}}^{\text{pop}}(\eta) = v^{\mathbf{ABCF},DE}(\eta) - v^{\mathbf{ABC},DEF}(\eta)$$

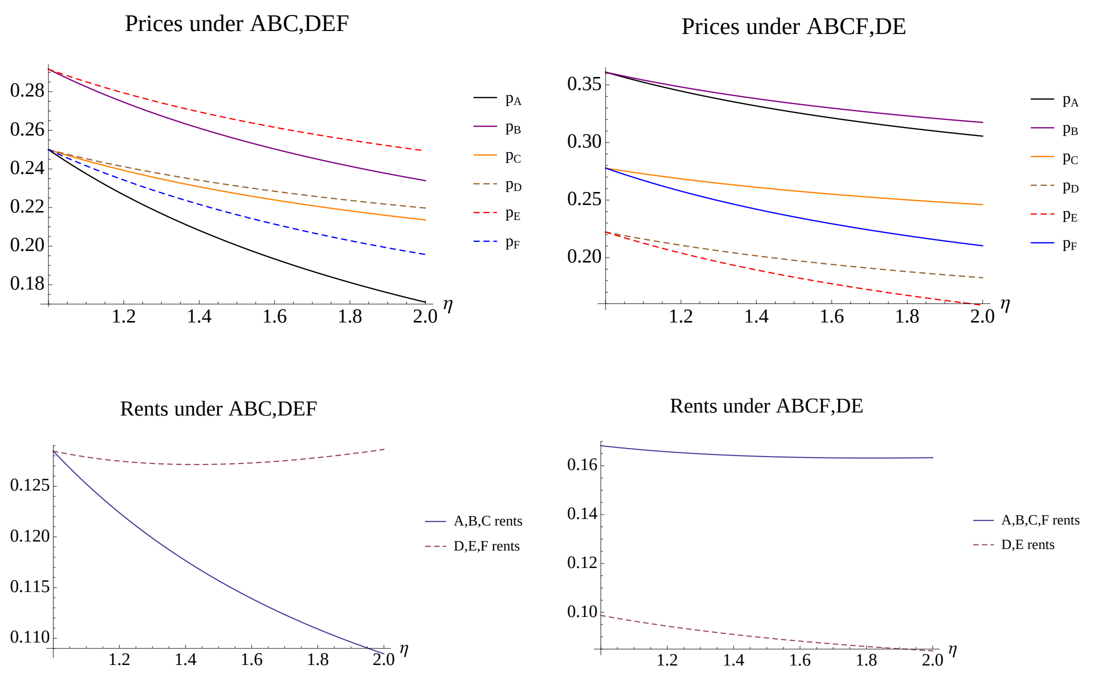

Figure 6.3: Prices and rents are decreasing in local population density (η) for both the ABC, DEF and $ABCF, DE$ configurations. The figures are drawn for the case of $t = 1/2$.

and the incremental return to winning for the defender is

$$\mathrm{IR}_{\mathrm{def}}^{\mathrm{pop}}(\eta) = v^{ABC,\mathbf{DEF}}(\eta) - v^{ABCF,\mathbf{DE}}(\eta).$$

Both factions want to win the conflict (i.e., both incremental returns are positive)—controlling territory F is valuable. Because of the direct effect of local population density at F on on demand at F, controlling territory F tends to decrease the rate at which a faction's rents decrease in local population density (indeed, for high enough η, rents can be increasing). That is, $v^{\mathbf{ABC},DEF}(\eta)$ is decreasing faster in η than is $v^{\mathbf{ABCF},DE}(\eta)$ (indeed, the latter is sometimes increasing) and $v^{ABCF,\mathbf{DE}}(\eta)$ is decreasing faster in η than is $v^{ABC,\mathbf{DEF}}(\eta)$ (again, the latter is sometimes increasing). Hence, for both factions, the incremental return to winning the conflict is increasing in η. This fact is illustrated in Figure 6.4 (and formalized in Proposition 6.2), where the left-hand cell shows the two components of each faction's incremental return and the right-hand cell shows the incremental returns themselves.

The attacker's incremental return is larger than the defender's and both faction's incremental returns are increasing in η. As highlighted in Section 3, expected violence is

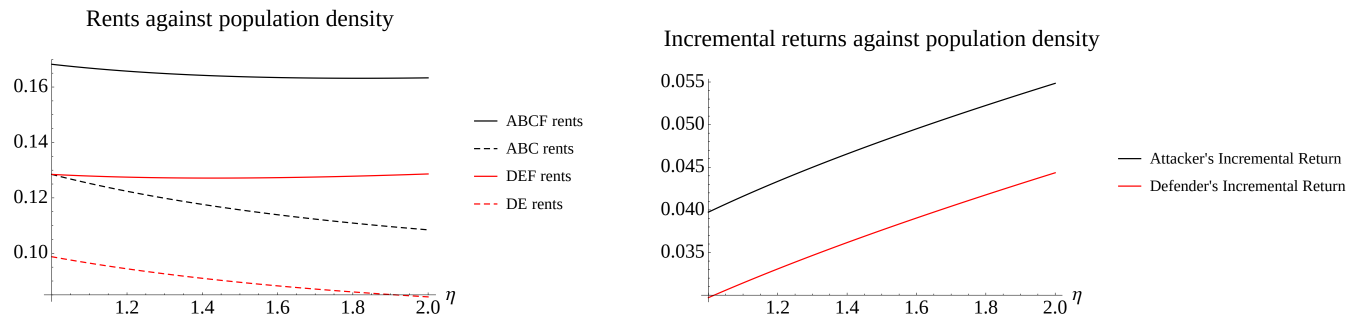

Figure 6.4: Rents are decreasing more slowly in population density for the faction that controls F. Hence, incremental returns are increasing in local population density. The figures are drawn for the case of $t = 1/2$.

increasing in the defender's incremental return and decreasing in the attacker's incremental return. So how does a change in local population density affect the expected amount of violence? As shown in Equation 2, the effect of the defender's incremental return dominates the effect of the attacker's incremental return, unless the attacker's incremental return changes a lot more than the defender's. This is clearly not the case here. Hence, overall, the effect of an increase in local population density is to increase incremental returns and expected violence.

Proposition 6.2 *Suppose there are two factions each of which controls three contiguous territories. Moreover, suppose the population in the* $1/6$*th of the country with the vulnerable territory at its center is of mass* ηN *for some* $\eta \in [1, 2]$*. The following are all true:*

(i) Regardless of what happens at the conflict stage:

- *Rents for the faction that does not end up with control of the vulnerable territory are decreasing in* η.
- *Rents for the faction that does end up with control of the vulnerable territory are decreasing in* η *at* $\eta = 1$*, are increasing in* η *at* $\eta = 2$*, and are strictly convex in* η.

(ii) The incremental returns to winning the conflict over a border territory are increasing in η *for both factions.*

(iii) The expected level of violence is increasing in η.

Proof. See Appendix B.3. ■

Here the model returns two surprising result in light of the standard intuitions. First, an increase in the size of a local market can be associated a decrease in profits for all factions. Second, even a change in local population density that is associated with a decrease in rents for both factions is associated with an increase in expected violence. Hence, again we see that rents and violence need not be positively correlated when the source of variation is changes to local conditions.

7 Number of Factions and Conflict

Having seen how both global and local features of the environment affect violence, I now turn to the configuration of territorial control itself. In particular, I query the effect of changes to the number of factions on violent outcomes. In order to hold all else equal while changing the number of factions, I compare configurations with symmetric factions. That is, I consider a configuration of territorial control with six factions each of which controls one territory, a configuration with three factions each of which controls two territories, and a configuration with two factions each of which controls three territories. I fix transportation costs and population density.

To characterize incremental returns, I first characterize equilibrium economic rents for all relevant configurations of territorial control. Table 7 provides a summary. The derivations of the associated equilibria are in Appendix A.

Configuration	Highest Payoff	2nd Highest Payoff	3rd Highest Payoff
$1,1,1,1,1,1$	$v^{1,1,1,1,1,1} = \frac{Nt}{36}$		
$2,1,1,1,1$	$v^{\mathbf{2},1,1,1,1} = \frac{145Nt}{2166}$	$v^{2,\mathbf{1},1,1,1} = \frac{40Nt}{1083}$	$v^{2,1,\mathbf{1},1,1} = \frac{100Nt}{3249}$
$2,2,2$	$v^{2,2,2} = \frac{Nt}{9}$		
$3,2,1$	$v^{\mathbf{3},2,1} = \frac{447{,}343Nt}{2{,}643{,}878}$	$v^{3,\mathbf{2},1} = \frac{298{,}831Nt}{2{,}643{,}876}$	$v^{3,2,\mathbf{1}} = \frac{5041Nt}{73{,}441}$
$3,3$	$v^{3,3} = \frac{37Nt}{144}$		
$4,2$	$v^{\mathbf{4},2} = \frac{109Nt}{324}$	$v^{4,\mathbf{2}} = \frac{16Nt}{81}$	

Table 7.1: Economic rents associated with different configurations of territorial control.

Six Factions Consider the game beginning with six factions. Suppose, to fix ideas, that territory A becomes vulnerable.

If the defender wins the fight, the outcome is the status quo. If the defender loses the fight, then it is eliminated, making a payoff of zero. Hence, the defender's incremental return is:

$$\mathrm{IR}_{\mathrm{def}}^{1,1,1,1,1,1} = v^{1,1,1,1,1,1} - 0.$$

There are two attackers—the factions in control of territories B and F. Since the two attackers are in symmetric situations, without loss of generality, focus on one attacker faction, say B. If the attacker wins the fight, it becomes the large faction in a five faction system. If it loses the fight, then one of two things happens: either the defender wins the fight and faction B is in a six faction configuration or the other attacker (F) wins the fight and B is one of the small factions bordering the large faction in a five faction system. Let π be faction B's belief about the probability of the defender (A) investing more than the other attacking faction (F). Then faction B's expected incremental return is:

$$\mathrm{IR}_{\mathrm{att}}^{1,1,1,1,1,1} = v^{\mathbf{2},1,1,1,1} - \pi v^{1,1,1,1,1,1} - (1-\pi)v^{2,\mathbf{1},1,1,1}.$$

Lemma 7.1 shows that the two attackers have strictly higher incremental returns to winning than does the defender, which is important for characterizing equilibrium play.

Lemma 7.1 $\mathrm{IR}_{\mathrm{att}}^{1,1,1,1,1,1} > \mathrm{IR}_{\mathrm{def}}^{1,1,1,1,1,1}$ *for any* $\pi \in [0,1]$.

Proof. See Appendix C ■

As established in Theorem 3.1, given that the two attacker's value the territory more than the defender, both attacker factions will play a mixed strategy with investment drawn from a uniform distribution on support $\left[0, \mathrm{IR}_{\mathrm{att}}^{1,1,1,1,1,1}\right]$. The defender invests zero and loses for certain, which implies that $\pi = 0$. At $\pi = 0$, we have

$$\mathrm{IR}_{\mathrm{att}}^{1,1,1,1,1,1}(\pi = 0) = v^{\mathbf{2},1,1,1,1} - v^{2,\mathbf{1},1,1,1} = \frac{65Nt}{2166}.$$

Given this, we have the following result:

Proposition 7.1 *When the initial configuration involves six factions, regardless of which territory becomes vulnerable, equilibrium play at the conflict stage is as follows:*

- *The defender faction invests zero and loses for certain.*
- *The attacker factions' investments are drawn independently from a uniform distribution on* $\left[0, \frac{65Nt}{2166}\right]$.

Consequently, the ex ante distribution of violence is as in Equation 1, with $\text{IR}_1^{1,1,1,1,1,1} = \text{IR}_2^{1,1,1,1,1,1} = \frac{65Nt}{2166}$.

Proof. Follows from Theorem 3.1, Equation 1, and the analysis in the text. ■

Three Equal Factions Now suppose there are three symmetric factions, each of which controls two contiguous territories. For any vulnerable territory, only two factions can fight—an attacker and a defender. The defender's incremental return is

$$\text{IR}_{\text{def}}^{2,2,2} = v^{2,2,2} - v^{3,2,\mathbf{1}} = \frac{28,072Nt}{660,969} \approx 0.0425Nt$$

The attacker's incremental return is

$$\text{IR}_{\text{att}}^{2,2,2} = v^{\mathbf{3},2,1} - v^{2,2,2} = \frac{51,193Nt}{881,292} \approx 0.0581Nt.$$

Hence, the equilibrium outcomes follow exactly from the analysis in Section 3.

Proposition 7.2 *When the initial configuration involves three equal factions, each of which controls two contiguous territories, regardless of which territory becomes vulnerable, equilibrium at the conflict stage is as follows:*

- *With probability* $\frac{112,288}{153,579}$*, the defender's investment is drawn from a uniform distribution on* $\left[0, \frac{28,072Nt}{660,969}\right]$ *and with complementary probability the defender invests zero.*
- *The attacker faction's investment is drawn independently from a uniform distribution on* $\left[0, \frac{28,072Nt}{660,969}\right]$.

The ex ante distribution of violence is as in Equation 1, with $\text{IR}_1^{2,2,2} = \frac{51,193Nt}{881,292}$ *and* $\text{IR}_2^{2,2,2} = \frac{28,072Nt}{660,969}$.

Proof. Follows from Theorem 3.1, Equation 1, and the analysis in the text. ■

Two Equal Factions The case of two equal factions was analyzed in Section 5 and equilibrium play is characterized in Proposition 5.1.

Given these characterizations, we can now assess the effect of a change in the number of factions on violent outcomes.

7.1 Factionalization and Violent Outcomes

How does a change in the number of factions affect violent outcomes? I decompose the analysis into four interrelated outcomes:

(i) The frequency of violence—i.e., the probability that the realization of f is positive.

(ii) The expected intensity of violence—i.e., the expectation of f conditional on its realization being positive.

(iii) The variability of violence—i.e., the variance of f.

(iv) The expected level of violence—i.e..e, the unconditional expectation of f.

Frequency of Violence Two factors affect how frequently violence occurs. The first is how often the vulnerable territory is ceded by all but one faction. The second is the percentage of safe territories—i.e., territories that only border other territories controlled by the same faction. Let's see how the number of factions affects these.

First, consider the case of six factions. In this setting, there are no safe territories. Further, because control is diffuse, there are always two attackers, one on each side of the vulnerable territory. So, while the defender cedes the territory with certainty, neither attacker is willing to cede—there is always violence.

This is not the case when the factions further consolidate. Comparing the configuration with three factions to the configuration with two factions, we see that two things change. First, as the factions consolidate, the incremental returns go up, which affects how often the defender cedes the territory to the attacker. Second, the consolidation to two factions creates two safe territories, which reduces the opportunities for violence to occur at all. As formalized in Proposition 7.3, putting these effects together, factional consolidation is associated with less frequent violence.

Proposition 7.3 *The frequency with which violence occurs is decreasing as one moves from a configuration with six factions, to three factions each controlling two contiguous territories, to two factions each controlling three contiguous territories.*

Proof. See Appendix C ■

Intensity of Violence The frequency of violence occurring is increasing in the number of factions. But, conditional on it occurring at all, the opposite is true of the expected intensity of that violence. The expected intensity of violence is the expectation of f, conditional on at least two factions being active in the fight, which is simply IR_{def}. Because the economic model has increasing returns, greater consolidation is associated with larger incremental returns.

Proposition 7.4 *The expected intensity of violence—that is, the expectation of f conditional on at least two factions making positive investments—is increasing as one moves from a configuration with six factions, to three factions each controlling two contiguous territories, together two factions each controlling three contiguous territories.*

Proof. The conditional expectation of f is simply the incremental return of the faction that values winning the second most. From From Propositions 7.1, 7.2, and 5.1, those incremental returns are:

$$\text{IR}_{\text{att}}^{1,1,1,1,1,1} = \frac{65Nt}{2166}$$

$$\text{IR}_{\text{def}}^{2,2,2} = \frac{28,072Nt}{660,969}$$

and

$$\text{IR}_{\text{def}}^{3,3} = \frac{77Nt}{1296}.$$

It is straightforward that these are ordered as required. ■

Variance of Violence Increased factionalization leads to more frequent, but less intense, violence. This straightforwardly leads to a prediction about the variability of violence. The fewer factions, the more likely are both very low (zero) and very high levels of violence. Consequently, consolidation is associated with greater variance in violence.

Proposition 7.5 *The variance of violence—that is, the variance of f—is increasing as one moves from a configuration with six factions, to three factions each controlling two contiguous territories, to two factions each controlling three contiguous territories.*

Proof. See Appendix C. ■

Expected Level of Violence Given all of these effects, how does the overall expected level of violence respond to factional consolidation? As summarized in Table 7.1, the expected level of violence is non-monotone in the number of factions. The reasons are subtle, reflecting the competing effects on intensity and frequency. Two key points are worth noting.

Configuration	IR_1	IR_2	Expected Violence
$1,1,1,1,1,1$	$\frac{65Nt}{2166} \approx 0.0300Nt$	$\frac{65Nt}{2166} \approx 0.0300Nt$	$\frac{65Nt}{2166} \approx 0.0300Nt$
$2,2,2$	$\frac{51{,}193}{881{,}292} \approx 0.0581Nt$	$\frac{28{,}072Nt}{660{,}969} \approx 0.0425Nt$	$\frac{3{,}152{,}148{,}736Nt}{101{,}510{,}958{,}051} \approx 0.0311Nt$
$3,3$ (border vulnerable)	$\frac{103Nt}{1296} \approx 0.0795Nt$	$\frac{77Nt}{1296} \approx 0.059Nt$	$\frac{5929Nt}{133{,}488} \approx 0.0444Nt$
$3,3$ (interior vulnerable)	N/A	N/A	0
$3,3$ (ex ante)	N/A	N/A	$\frac{2}{3} \times \frac{5929Nt}{133{,}488} \approx 0.0296Nt$

Table 7.2: Expected violence as a function of the number of factions and vulnerability.

First, the expected level of violence, conditional on a border region being vulnerable, is monotonically decreasing in the number of factions. This monotonicity is not straightforward, since there are effects on both the frequency and intensity of violence which cut in opposite directions. So why is this the case?

The incremental returns to winning for both attackers and defenders are increasing as territorial control becomes more concentrated. This generates competing effects on the expected level of violence. As shown in Equation 2, the positive effect of the increase in IR_2 dominates the negative effect of the increase in IR_1 unless IR_1 increases more than twice as much as IR_2, which is not the case here. Hence, expected violence is monotonically increasing as the number of factions decreases.

But now consider the ex ante expected level of fighting without conditioning on a border region being vulnerable. There is an additional effect that creates the non-monotonicity. The consolidation to two factions creates two safe territories so that no conflict is possible one-third of the time. Overall, then, equilibrium incentives for investing in violence increase as the number of factions decreases, but opportunities for conflict decrease. Hence, from an ex ante perspective, the scenario with only two factions has the lowest level of expected violence even though, conditional on a border region becoming vulnerable, it has the highest level of expected violence.

Proposition 7.6 *The expected level of violence—that is, the expectation of f—is non-monotone as one moves from a configuration with six factions, to three factions each controlling two contiguous territories, to two factions each controlling three contiguous territories. In particular,*

$$\mathbb{E}[f|2,2,2] > \mathbb{E}[f|1,1,1,1,1,1] > \mathbb{E}[f|3,3].$$

Proof. Follows from the argument in the text and Table 7.1. ■

7.2 Factionalization and Stability

Factional consolidation has a variety of effects on the distribution of violence—decreasing the frequency of violence, increasing the intensity and variance of violence, and affecting expected violence non-monotonically. Given all of this, it is natural to ask whether more consolidated configurations are more or less stable.

As we've already seen, the most highly factionalized environment in this model is special. In that environment, because there are attackers on both sides, the vulnerable territory always changes hands—the defender cedes the territory to the two attackers, who fight over it, each winning with probability one-half.

Matters are more complicated in more consolidated configurations. In particular, there are now two forces at work, pushing in opposite directions.

First, moving from three to two factions increases scare-off—i.e., conditional on a border territory being vulnerable, the defender is more likely to cede when there are only two factions. This is because consolidation makes the attacker's incremental return increase faster than the defender's. One can see this by comparing the probability of ceding in each configuration:

$$1 - \frac{\mathrm{IR}^{2,2,2}_{\mathrm{def}}}{\mathrm{IR}^{2,2,2}_{\mathrm{att}}} = \frac{41,291}{153,579} < \frac{26}{103} = 1 - \frac{\mathrm{IR}^{3,3}_{\mathrm{def}}}{\mathrm{IR}^{3,3}_{\mathrm{att}}}.$$

This effect is recorded in the second column of Table 7.2, which shows the probability that an attacker wins a conflict, given that a contestable region is vulnerable.

The second force has to do with safe territory. More highly concentrated territorial control creates safe territories that are not subject to capture. Safe territories increase stability.

As shown in the third column of Table 7.2, these two effects net out such that the stability is increasing in factional concentration—i.e., the fewer factions, the less likely there is to be a change of territorial control.

Proposition 7.7 *The stability of the configuration of territorial control is increasing as one moves from a configuration with six factions, to three factions each controlling two contiguous territories, to two factions each controlling three contiguous territories.*

Proof. Follows from the argument in the text and Table 7.2. ■

Configuration	Probability Attacker Wins Contestable Region	Overall Transition Probability
$1, 1, 1, 1, 1, 1$	1	1
$2, 2, 2$	$\frac{97,435}{153,579} \approx 0.634$	$\frac{97,435}{153,579} \approx 0.634$
$3, 3$	$\frac{129}{206} \approx 0.626$	$\frac{2}{3} \times \frac{129}{206} \approx 0.417$

Table 7.3: Probability a region changes hands as a function of the number of factions and vulnerability.

8 Conclusion

I study a model of armed factions fight over control of territory from which they endogenously extract economic rents. The analysis, building on canonical models of both conflict and spatial price competition, yields several results worth reemphasizing.

First, how changes to market features affect the distribution of violence depends on whether those changes are local or global. Most of the modern empirical literature exploits local variation. Yet, the model's predictions about the effects of local changes are different from the conventional hypotheses (which are more similar to the models' results regarding global changes). In particular, the model predicts that changes to local market conditions (transportation costs or population density) that increase the rents associated with controlling a particular territory lead to a decrease in violence.

Second, qualitative accounts and conventional wisdom suggest that an increase in the number of armed factions leads to an increase in violence. Here, the predicted relationship is more subtle. An increase in the number of factions does lead violence to break out more frequently—both by decreasing scare-off and by increasing opportunities for violence. However, when violence occurs, the fewer factions, the more intense it is. Highly factionalized environments, then, are characterized by frequent, low-level violence and instability of the pattern of territorial control. Consolidated environments are characterized by infrequent, high-level violence and stability of the pattern of territorial control. The overall expected amount of violence is non-monotone in the number of factions.

Finally, the model highlights a conceptual point. Typically, models of conflict have taken the returns to winning the conflict to be exogenous to conflict outcomes. The results here only arise because conflict outcomes feedback into economic behavior, which affect the returns to winning the conflict. Hence, the model demonstrates the importance of a political economy approach to the study of conflict which takes seriously this endogenous interaction between economic and conflict behavior.

A Economic Equilibrium

A.1 Six Factions

Suppose there are six factions, each of which controls one territory. If demand is characterized by Equation 3 at some vector of prices, faction i's profits are:

$$p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right].$$

Given the symmetry of the factions, equilibrium prices are characterized by the following condition:

$$N\left[\frac{2p^* - 2p^*}{2t} + \frac{1}{6}\right] - \frac{Np^*}{t} = 0.$$

This implies that in equilibrium, the common price is

$$p^*_{1,1,1,1,1,1} = \frac{t}{6}$$

Note that for any $t < 1$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ for all i, j, so demand is in fact characterized by Equation 3.

Each faction's equilibrium rents are

$$v^{1,1,1,1,1,1} = \frac{t}{6} \times \frac{N}{6} = \frac{Nt}{36}.$$

A.2 Five Factions

Suppose there are five factions—so one faction controls two regions and all the remaining factions control one. Without loss of generality, suppose the large faction controls regions A and B. The there are three kinds of factions to consider:

(i) Large faction (controls A and B)

(ii) Border faction (controls C or F)

(iii) Interior faction (controls D or E)

If demand is characterized by Equation 3 at some vector of prices, the large faction's profits are:

$$N\left[p_A\left(\frac{1}{6} + \frac{p_B + p_F - 2p_A}{2t}\right) + p_B\left(\frac{1}{6} + \frac{p_A + p_C - 2p_B}{2t}\right)\right],$$

the C-border faction's profits are:

$$Np_C\left(\frac{1}{6}+\frac{p_B+p_D-2p_C}{2t}\right),$$

and the $D-$interior faction's profits are:

$$Np_D\left(\frac{1}{6}+\frac{p_C+p_E-2p_D}{2t}\right).$$

An equilibrium can be described by the following first-order and symmetry conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_B^*+p_D^*-2p_C^*}{2t}-\frac{p_C^*}{t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_E^*-2p_D^*}{2t}-\frac{p_D^*}{t}=0$$

$$p_A^*=p_B^*$$

$$p_C^*=p_F^*$$

$$p_D^*=p_E^*.$$

This implies that in equilibrium, we have:

$$p_A^*=p_B^*=\frac{5t}{19}$$

$$p_C^*=p_F^*=\frac{11t}{57}$$

$$p_D^*=p_E^*=\frac{10t}{57}.$$

Note that for any $t<1$, we have $p_i\leq 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact characterized by Equation 3.

The large faction's profits are

$$v^{\mathbf{2},1,1,1,1}=\frac{145Nt}{2166},$$

the border factions' profits are

$$v^{2,\mathbf{1},1,1,\mathbf{1}}=\frac{40Nt}{1083},$$

and the interior factions' profits are:

$$v^{2,1,\mathbf{1},1} = \frac{100Nt}{3249}.$$

A.3 Three Equal Factions

Suppose there are three factions, each controlling two contiguous territories. If demand is characterized by Equation 3 at some vector of prices, then a faction controlling territories i and $i+1$ has profits:

$$p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right] + p_{i+1} \left[D_{i+1}(p_{i+1}, p_{i+2}) + D_{i+1}(p_{i+1}, p_i)\right].$$

Given the symmetry of the factions, equilibrium prices are described by the following condition:

$$\frac{1}{6} - \frac{p^*}{t} + \frac{p^*}{2t} = 0,$$

which implies the following common price:

$$p^*_{2,2,2} = \frac{t}{3}$$

Notice $p^*_{2,2,2} > 2 - p^*_{2,2,2} - \frac{t}{6}$ for any $t < 1$, so demand is in fact characterized by Equation 3.

Equilibrium profits are:

$$v^{2,2,2} = \frac{t}{3} \times \frac{N}{3} = \frac{Nt}{9}.$$

A.4 Three Unequal Factions

Without loss of generality, suppose the three factions are ABC, DE, F.

The large faction's payoffs are

$$N\left[p_A\left(\frac{1}{6} + \frac{p_B + p_F - 2p_A}{2t}\right) + p_B\left(\frac{1}{6} + \frac{p_A + p_C - 2p_B}{2t}\right) + p_C\left(\frac{1}{6} + \frac{p_B + p_D - 2p_C}{2t}\right)\right],$$

the medium faction's payoffs are

$$N\left[p_D\left(\frac{1}{6} + \frac{p_C + p_E - 2p_D}{2t}\right) + p_E\left(\frac{1}{6} + \frac{p_D + p_F - 2p_E}{2t}\right)\right],$$

and the small faction's payoffs are

$$Np_F\left(\frac{1}{6}+\frac{p_E+p_A-2p_F}{2t}\right).$$

Prices satisfy the following six first-order conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_B^*+p_D^*-2p_C^*}{2t}-\frac{p_C^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_E^*-2p_D^*}{2t}-\frac{p_D^*}{t}+\frac{p_E^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_D^*+p_F^*-2p_E^*}{2t}-\frac{p_E^*}{t}+\frac{p_D^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_A^*+p_E^*-2p_F^*}{2t}-\frac{p_F^*}{t}=0$$

Solving, this implies the following equilibrium prices:

$$p_A^*=\frac{637t}{1626}\quad p_B^*=\frac{395t}{813}\quad p_C^*=\frac{112t}{271}$$

$$p_D^*=\frac{283t}{813}\quad p_E^*=\frac{175t}{542}$$

$$p_F^*=\frac{71t}{271}.$$

Note that for any $t<1$, we have $p_i \leq 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact characterized by Equation 3.

These prices imply the following equilibrium profits:

$$v^{\mathbf{3},2,1}=\frac{447,343Nt}{2,643,878}$$

$$v^{3,\mathbf{2},1}=\frac{298,831Nt}{2,643,876}$$

$$v^{3,2,\mathbf{1}}=\frac{5041Nt}{73,441}.$$

A.5 Two Equal Factions

Suppose there are two factions, each controlling three contiguous territories. Without loss of generality, suppose the factions control A, B, C and D, E, F, respectively.

If demand is characterized by Equation 3 at some vector of prices, then a faction controlling territories $i-1$, i and $i+1$ has profits:

$$p_{i-1}\left[D_{i-1}(p_{i-1},p_i)+D_{i-1}*p_{i-1},p_{i-2})\right]+p_i\left[D_i(p_i,p_{i+1})+D_i(p_i,p_{i-1})\right]+p_{i+1}\left[D_{i+1}(p_{i+1},p_{i+2})+D_{i+1}(p_{i+1},p_i)\right].$$

Equilibrium prices are described by the following first-order and symmetry conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$

$$p_A^*=p_C^*=p_D^*=p_F^*$$

$$p_B^*=p_E^*.$$

Solving, this implies the following equilibrium prices:

$$p_A^*=p_C^*=p_D^*=p_F^*=\frac{t}{2}$$

$$p_B^*=p_E^*=\frac{7t}{12}.$$

Note, $p_i \leq 2-p_j-\frac{t}{6}$ for all i, j, so demand is in fact described by Equation 3.

Equilibrium profits for each faction are:

$$v^{3,3}=\frac{37Nt}{144}.$$

A.6 Two Moderately Unequal Factions

Suppose there are two factions, one controlling four contiguous territories and one controlling two contiguous territories. Without loss of generality, suppose the two factions control A, B, C, D and E, F.

The large faction's payoffs are:

$$N\left(p_A\left(\frac{1}{6}+\frac{p_B+p_F-2p_A}{2t}\right)+p_B\left(\frac{1}{6}+\frac{p_A+p_C-2p_B}{2t}\right)+p_C\left(\frac{1}{6}+\frac{p_B+p_D-2p_C}{2t}\right)+p_D\left(\frac{1}{6}+\frac{p_C+p_E-2p_D}{2t}\right)\right),$$

and the small faction's payoffs are:

$$N\left(p_E\left(\frac{1}{6}+\frac{p_D+p_F-2p_E}{2t}\right)+p_F\left(\frac{1}{6}+\frac{p_E+p_A-2p_F}{2t}\right)\right).$$

In equilibrium, prices are described by the following first-order and symmetry conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_A^*+p_E^*-2p_F^*}{2t}-\frac{p_F^*}{t}+\frac{p_E^*}{2t}=0$$

$$p_A^*=p_D^*$$

$$p_B^*=p_C^*$$

$$p_E^*=p_F^*.$$

Solving, this implies the following equilibrium prices:

$$p_A^*=p_D^*=\frac{5t}{9}$$

$$p_B^*=p_C^*=\frac{13t}{18}$$

$$p_E^*=p_F^*=\frac{4t}{9}.$$

Note, $p_i \leq 2-p_j-\frac{t}{6}$ for all i, j, so demand is in fact described by Equation 3.

These prices imply the following profits:

$$v^{\mathbf{4},2}=\frac{109Nt}{324}$$

$$v^{4,\mathbf{2}}=\frac{16Nt}{81}.$$

A.7 Two Highly Unequal Factions

Suppose there are two factions, one controlling five territories and the other controlling one territory. Without loss of generality, suppose the two factions control A, B, C, D, E and F.

The large faction's payoffs are:

$$N\Bigg[p_A\left(\frac{1}{6}+\frac{p_B+p_F-2p_A}{2t}\right)+p_B\left(\frac{1}{6}+\frac{p_A+p_C-2p_B}{2t}\right)+p_C\left(\frac{1}{6}+\frac{p_B+p_D-2p_C}{2t}\right)$$
$$+p_D\left(\frac{1}{6}+\frac{p_C+p_E-2p_D}{2t}\right)+p_E\left(\frac{1}{6}+\frac{p_D+p_F-2p_E}{2t}\right)\Bigg],$$

and the small faction's payoffs are:

$$Np_F\left(\frac{1}{6}+\frac{p_E+p_A-2p_F}{2t}\right).$$

In equilibrium, prices are described by the following first-order and symmetry conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$
$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$
$$\frac{1}{6}+\frac{p_D^*+p_B^*-2p_C^*}{2t}-\frac{p_C^*}{t}+\frac{p_B^*}{2t}+\frac{p_D^*}{2t}=0$$
$$\frac{1}{6}+\frac{p_A^*+p_E^*-2p_F^*}{2t}-\frac{p_F^*}{t}=0$$
$$p_A^*=p_E^*$$
$$p_B^*=p_D^*.$$

Solving, this implies the following equilibrium prices:

$$p_A^*=p_E^*=\frac{11t}{18}$$
$$p_B^*=p_D^*=\frac{31t}{36}$$
$$p_C^*=\frac{17t}{18}$$
$$p_F^*=\frac{7t}{18}.$$

Note, $p_i \leq 2-p_j-\frac{t}{6}$ for all i, j, so demand is in fact described by Equation 3.

Profits are:

$$v^{\mathbf{5},1}=\frac{287Nt}{648}$$

$$v^{5,\mathbf{1}} = \frac{49Nt}{324}.$$

B Comparative statics

B.1 Global Comparative Statics

Proof of Proposition 5.2. Expected violence is

$$\frac{2}{3} \times \frac{77}{103} \times \frac{77Nt}{1296},$$

which is obviously increasing in N and t.

Now consider the variance. An arbitrary random variable whose distribution places mass α on zero and mass $1-\alpha$ on a draw from a symmetric triangular distribution on $[0, b]$ has variance:

$$\frac{(1 + 5\alpha - 6\alpha^2)b^2}{24}. \tag{7}$$

Violence, here, is such a random variable, with

$$\alpha = \frac{1}{3} + \frac{2}{3}\left(1 - \frac{\mathrm{IR}^{3,3}_{\mathrm{def}}}{\mathrm{IR}^{3,3}_{\mathrm{att}}}\right)$$

and

$$b = 2\mathrm{IR}^{3,3}_{\mathrm{def}}.$$

Plugging these in to Equation 7 yields the following variance:

$$\sigma^2_{3,3} = \frac{188,548,129N^2t^2}{160,371,415,296},$$

which is increasing in N and t.

The probability of transitioning from $3,3$ to $4,2$ is

$$\frac{2}{3} \times \frac{77}{103} \times \frac{1}{2},$$

which is constant in t and N. ■

B.2 Local Transportation Costs

Without loss of generality, suppose the two factions start controlling A, B, C and D, E, F. To find the incremental returns, I start by characterizing equilibrium in the two scenarios: ABC, DEF and $ABCF, DE$

B.2.1 ABC, DEF

Taking first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(62\tau^2 + 281\tau + 197\right)t}{18\left(2\tau^2 + 19\tau + 39\right)}$$

$$p_B = \frac{\left(106\tau^2 + 571\tau + 583\right)t}{36(\tau+3)(2\tau+13)}$$

$$p_C = \frac{\left(38\tau^2 + 233\tau + 269\right)t}{18(\tau+3)(2\tau+13)}$$

$$p_D = \frac{\left(34\tau^2 + 247\tau + 259\right)t}{18(\tau+3)(2\tau+13)}$$

$$p_E = \frac{(43\tau + 41)t}{36(\tau+3)}$$

$$p_F = \frac{\left(40\tau^2 + 259\tau + 241\right)t}{18(\tau+3)(2\tau+13)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equation 5.

The order of prices is $p_E > p_B > p_A > p_F > p_D > p_C$. Hence, there are two candidates for the worst-off citizen: the citizen indifferent between buying from E and F and the citizen indifferent between buying from A and B.

The citizen indifferent between E and F is located at x^*_{EF} satisfying:

$$p_E + x^*_{EF} t = p_F + \left(\frac{1}{6} - x^*_{EF}\right)\tau t$$

or

$$x^*_{EF} = \frac{4\tau^3 + 36\tau^2 + 37\tau - 17}{24\tau^3 + 252\tau^2 + 696\tau + 468}.$$

We need the following:

$$1 - p_E - x^*_{EF} t \geq 0$$

which is true if

$$\frac{-98\tau^3 t + 72\tau^3 - 835\tau^2 t + 756\tau^2 - 1285\tau t + 2088\tau - 482t + 1404}{36(\tau+3)\left(2\tau^2 + 15\tau + 13\right)} \geq 0$$

The LHS of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality reduces to:

$$\frac{-26\tau^3 - 79\tau^2 + 803\tau + 922}{36(\tau+3)\left(2\tau^2 + 15\tau + 13\right)} \geq 0,$$

which is true for any $\tau \in [1, 2]$.

The citizen indifferent between A and B is located at x^*_{AB} satisfying:

$$p_A + x^*_{AB} t = p_B + \left(\frac{1}{6} - x^*_{AB}\right) t$$

or

$$x^*_{AB} = \frac{47 - 2\tau}{48\tau + 312}.$$

We need the following:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if

$$\frac{-242\tau^2 t + 144\tau^2 - 1247\tau t + 1368\tau - 1211t + 2808}{72(\tau+3)(2\tau+13)}$$

The LHS of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality reduces to:

$$\frac{-98\tau^2 + 121\tau + 1597}{72(\tau+3)(2\tau+13)} \geq 0$$

which is true for any $\tau \in [1, 2]$.

The rents at these equilibrium prices are:

$$v^{\mathbf{ABC},DEF}(\tau) = \frac{\left(3500\tau^4 + 46780\tau^3 + 190407\tau^2 + 252436\tau + 106277\right) Nt}{1296(\tau+1)(\tau+3)(2\tau+13)^2}$$

and

$$v^{ABC,\mathbf{DEF}}(\tau) = \frac{\left(4948\tau^4 + 75452\tau^3 + 351465\tau^2 + 520802\tau + 246133\right) Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2}.$$

B.2.2 $ABCF, DE$

Taking first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(62\tau^2 + 376\tau + 303\right) t}{9\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_B = \frac{\left(212\tau^2 + 1351\tau + 1401\right) t}{36\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_C = \frac{19(2\tau + 3)t}{9(4\tau + 15)}$$

$$p_D = \frac{\left(68\tau^2 + 415\tau + 429\right) t}{18\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_E = \frac{\left(43\tau^2 + 239\tau + 174\right) t}{9\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_F = \frac{(179\tau + 201)t}{36(4\tau + 15)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equation 5.

The order of prices is $p_A > p_B > p_F > p_C > p_E > p_D$. Hence, there are two candidates for the worst-off citizen: the citizen indifferent between buying from A and F and the citizen indifferent between buying from A and B.

The citizen indifferent between A and F is located at x^*_{AF} satisfying:

$$p_A + x^*_{AF} t = p_F + \left(\frac{1}{6} - x^*_{AF}\right) \tau t$$

or

$$x^*_{AF} = \frac{8\tau^3 + 47\tau^2 + 14\tau - 69}{48\tau^3 + 468\tau^2 + 1320\tau + 900}.$$

We need the following:

$$1 - p_A - x^*_{AF} t \geq 0$$

which is true if

$$\frac{16\tau^3 + 915\tau^2 + 5162\tau + 4395}{72(\tau + 1)\left(4\tau^2 + 35\tau + 75\right)} \geq 0,$$

which is clearly true for any $\tau \in [1, 2]$.

The citizen indifferent between A and B is located at x_{AB}^* satisfying:

$$p_A + x_{AB}^* t = p_B + \left(\frac{1}{6} - x_{AB}^*\right) t$$

or

$$x_{AB}^* = \frac{-4k^2 + 19k + 213}{96k^2 + 840k + 1800}.$$

We need the following:

$$1 - p_A - x_{AB}^* t \geq 0$$

which is true if

$$\frac{-484\tau^2 t + 288\tau^2 - 3065\tau t + 2520\tau - 3063t + 5400}{72\left(4\tau^2 + 35\tau + 75\right)} \geq 0.$$

The LHS of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality reduces to:

$$\frac{-196\tau^2 - 545\tau + 2337}{72\left(4\tau^2 + 35\tau + 75\right)} \geq 0$$

which is true for any $\tau \in [1, 2]$.

The rents at these equilibrium prices are:

$$v^{\mathbf{ABCF},DE}(\tau) = \frac{\left(14000\tau^4 + 158266\tau^3 + 582603\tau^2 + 782964\tau + 350919\right) Nt}{1296(\tau+1)(\tau+5)(4\tau+15)^2}$$

and

$$v^{ABCF,\mathbf{DE}}(\tau) = \frac{\left(2474\tau^4 + 26854\tau^3 + 93111\tau^2 + 111528\tau + 43281\right) Nt}{324(\tau+1)(\tau+5)(4\tau+15)^2}.$$

Taken together, when a border region is vulnerable, the incremental returns are:

$$\begin{aligned} \mathrm{IR}_{\text{att}}^{\text{trans}}(\tau) &= v^{\mathbf{ABCF},DE}(\tau) - v^{\mathbf{ABC},DEF}(\tau) \\ &= \frac{\left(14000\tau^4 + 158266\tau^3 + 582603\tau^2 + 782964\tau + 350919\right) Nt}{1296(\tau+1)(\tau+5)(4\tau+15)^2} - \frac{\left(3500\tau^4 + 46780\tau^3 + 190407\tau^2 + 252436\tau + 106277\right) Nt}{1296(\tau+1)(\tau+3)(2\tau+13)^2} \end{aligned}$$

and

$$\begin{aligned} \mathrm{IR}_{\text{def}}^{\text{pop}}(\eta) &= v^{ABC,\mathbf{DEF}}(\eta) - v^{ABCF,\mathbf{DE}}(\eta) \\ &= \frac{\left(4948\tau^4 + 75452\tau^3 + 351465\tau^2 + 520802\tau + 246133\right) Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2} - \frac{\left(2474\tau^4 + 26854\tau^3 + 93111\tau^2 + 111528\tau + 43281\right) Nt}{324(\tau+1)(\tau+5)(4\tau+15)^2}. \end{aligned}$$

Proof of Proposition 6.1.

(i) Differentiating the rents, we have

$$\frac{\partial v^{\mathbf{ABC},DEF}(\tau)}{\partial \tau} = \frac{\left(6360\tau^5 + 108628\tau^4 + 663482\tau^3 + 1800891\tau^2 + 2005820\tau + 760819\right) Nt}{324(\tau+1)^2(\tau+3)^2(2\tau+13)^3}$$

$$\frac{\partial v^{ABC,\mathbf{DEF}}(\tau)}{\partial \tau} = \frac{\left(8664\tau^5 + 203140\tau^4 + 1567538\tau^3 + 5015871\tau^2 + 5991404\tau + 2279383\right) Nt}{1296(\tau+1)^2(\tau+3)^2(2\tau+13)^3}$$

$$\frac{\partial v^{\mathbf{ABCF},DE}(\tau)}{\partial \tau} = \frac{\left(61468\tau^5 + 1026583\tau^4 + 6237580\tau^3 + 16551342\tau^2 + 17968716\tau + 6551415\right) Nt}{648(\tau+1)^2(\tau+5)^2(4\tau+15)^3}$$

$$\frac{\partial v^{ABCF,\mathbf{DE}}(\tau)}{\partial \tau} = \frac{\left(13090\tau^5 + 212431\tau^4 + 1270000\tau^3 + 3351690\tau^2 + 3660714\tau + 1369035\right) Nt}{162(\tau+1)^2(\tau+5)^2(4\tau+15)^3}$$

all of which are clearly positive for $\tau \in [1, 2]$.

(ii) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}_{\mathrm{att}}^{\mathrm{trans}}(\tau)}{\partial \tau} = \frac{-Nt}{648(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3}$$
$$\Big[322336\tau^{10}+10451448\tau^9+143061988\tau^8+1073013626\tau^7+4766618725\tau^6+12523786196\tau^5$$
$$+ 17710031949\tau^4 + 8367954734\tau^3 - 7878111669\tau^2 - 8896414788\tau - 1152922545\Big]$$

and

$$\frac{\partial \mathrm{IR}_{\mathrm{def}}^{\mathrm{trans}}(\tau)}{\partial \tau} = \frac{-Nt}{1296(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3}$$
$$\Big[283264\tau^{10}+10174464\tau^9+163482400\tau^8+1529546792\tau^7+9107162500\tau^6+35555048270\tau^5$$
$$+90894354783\tau^4+148662284540\tau^3+149453302806\tau^2+87077604366\tau+24236491815\Big]$$

The incremental returns are decreasing if the arguments in square brackets are positive. This is clearly the case for the defender. Now consider the attacker. Here, notice that, for any $\tau \in [1, 2]$, the following are true:

$$8367954734\tau^3 > 1152922545$$

$$17710031949\tau^4 > 8896414788\tau$$

$$12523786196\tau^5 > 7878111669\tau^2$$

so all of the negative terms are more than off-set by positive terms.

(iii) In the event that an interior territory is vulnerable, violence is zero. Hence, it suffices to focus on the case of a border territory being vulnerable.

First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, this is the case if:

$$\mathrm{IR}_{\mathrm{att}}^{\mathrm{trans}}(\tau)-\mathrm{IR}_{\mathrm{def}}^{\mathrm{pop}}(\eta)=\frac{\left(90352\tau^5+1713756\tau^4+11545728\tau^3+33407975\tau^2+39254262\tau+15356727\right)Nt}{2592(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2}>0,$$

which clearly holds for any $\tau \in [1,2]$.

Thus, expected violence is

$$\frac{\mathrm{IR}_{\mathrm{def}}^{\mathrm{trans}}(\tau)^2}{\mathrm{IR}_{\mathrm{att}}^{\mathrm{trans}}(\tau)}=\frac{\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)^2Nt}{10368(\tau+1)(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial\tau}\frac{\mathrm{IR}_{\mathrm{def}}^{\mathrm{trans}}(\tau)^2}{\mathrm{IR}_{\mathrm{att}}^{\mathrm{trans}}(\tau)}=-\left(\frac{\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)Nt}{10368(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154)^2}\right)\times$$
$$\Big[22826546176\tau^{16}+1346834559616\tau^{15}+37276519674400\tau^{14}+639371782994576\tau^{13}+$$
$$7567435768222208\tau^{12}+65199087795895376\tau^{11}+420975308247002594\tau^{10}+2069002610638570577\tau^9+$$
$$7793137617277828811\tau^8+22498728719469456958\tau^7+49489440661438539010\tau^6+$$
$$81914683489662021400\tau^5+99928825843407467628\tau^4+86905973146295199618\tau^3+$$
$$50946917644932029964\tau^2+18077056655295975543\tau+2945458294230415545\Big]$$

which is clearly negative.

■

B.3 Local Population Density

Without loss of generality, suppose the two factions control start controlling A, B, C and D, E, F. To find the incremental returns, I start by characterizing equilibrium in the two scenarios: ABC, DEF and $ABCF, DE$

B.3.1 ABC, DEF

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(16\eta^2 + 94\eta + 25\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_B = \frac{\left(86\eta^2 + 185\eta + 44\right) t}{36\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_C = \frac{\left(46\eta^2 + 73\eta + 16\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_D = \frac{\left(50\eta^2 + 71\eta + 14\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_E = \frac{\left(106\eta^2 + 175\eta + 34\right) t}{36\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_F = \frac{\left(32\eta^2 + 86\eta + 17\right) t}{18\left(8\eta^2 + 6\eta + 1\right)}.$$

These prices are consistent with $p_A < p_F$ and $p_E \geq p_F$ for any $\eta \in [1, 2]$. Hence, this case is a candidate for an equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(11\eta^2 + 74\eta + 50\right) t}{9\left(11\eta^2 + 15\eta + 4\right)}$$

$$p_B = \frac{\left(113\eta^2 + 341\eta + 176\right) t}{36(\eta + 1)(11\eta + 4)}$$

$$p_C = \frac{\left(29\eta^2 + 74\eta + 32\right) t}{9(\eta + 1)(11\eta + 4)}$$

$$p_D = \frac{\left(53\eta^2 + 161\eta + 56\right) t}{18(\eta + 1)(11\eta + 4)}$$

$$p_E = \frac{(4\eta + 17)t}{18(\eta + 1)}$$

$$p_F = \frac{\left(44\eta^2 + 161\eta + 65\right) t}{18(\eta + 1)(11\eta + 4)}.$$

These prices are inconsistent with $p_E < p_F$, so there is no such equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(13\eta^2 + 97\eta + 25\right) t}{9(23\eta + 7)}$$

$$p_B = \frac{(2\eta + 19)t}{36}$$

$$p_C = \frac{\left(10\eta^2 + 94\eta + 31\right) t}{9(23\eta + 7)}$$

$$p_D = \frac{\left(34\eta^2 + 163\eta + 73\right) t}{18(23\eta + 7)}$$

$$p_E = \frac{\left(58\eta^2 + 163\eta + 94\right) t}{18(23\eta + 7)}$$

$$p_F = \frac{\left(58\eta^2 + 187\eta + 25\right) t}{18(23\eta + 7)}.$$

These prices are inconsistent with $p_E < p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(2\eta + 13)t}{30}$$

$$p_B = \frac{(2\eta + 19)t}{36}$$

$$p_C = \frac{(4\eta + 41)t}{90}$$

$$p_D = \frac{(13 + 2\eta)t}{30}$$

$$p_E = \frac{(4\eta + 17)t}{36}$$

$$p_F = \frac{(14\eta + 31)t}{90}.$$

These prices are inconsistent with $p_A \geq p_F$, so there is no such equilibrium.

We have only one candidate for an equilibrium (case (i)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equation 6. In this candidate profile of prices, the prices are ordered as follows:

$$p_E > p_B > p_D > p_C > p_F > p_A.$$

Hence, the worst-off population member is the one who is just indifferent between buying from D and E. This person is located at x^*_{DE} given by:

$$p_D + x^*_{DE}t = p_E + \left(\frac{1}{6} - x^*_{DE}\right)t$$

or

$$x^*_{DE} = \frac{18\eta^2 + 23\eta + 4}{192\eta^2 + 144\eta + 24}.$$

This person prefers to buy the good as long as:

$$1 - p_D - x^*_{DE}t \geq 0.$$

This is true if and only if:

$$\frac{-254\eta^2 t + 576\eta^2 - 353\eta t + 432\eta - 68t + 72}{72\left(8\eta^2 + 6\eta + 1\right)} \geq 0.$$

The LHS is linearly decreasing in t, so it suffices to check $t = 1$. For $t = 1$, the inequality holds if

$$322\eta^2 + 79\eta + 4 \geq 0,$$

which is always the case.

Equilibrium rents are:

$$v^{\mathbf{ABC},DEF}(\eta) = \frac{(602 + 6408\eta + 23371\eta^2 + 32308\eta^3 + 11724\eta^4 + 512\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}$$

and

$$v^{ABC,\mathbf{DEF}}(\eta) = \frac{(410 + 4752\eta + 19315\eta^2 + 31492\eta^3 + 16908\eta^4 + 2048\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}.$$

B.3.2 $ABCF, DE$

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(128\eta^2 + 372\eta + 241\right) t}{18(46\eta + 11)}$$

$$p_B = \frac{\left(92\eta^2 + 441\eta + 208\right) t}{18(46\eta + 11)}$$

$$p_C = \frac{\left(28\eta^2 + 186\eta + 71\right) t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(20\eta^2 + 165\eta + 43\right) t}{9(46\eta + 11)}$$

$$p_E = \frac{2\left(13\eta^2 + 84\eta + 17\right) t}{9(46\eta + 11)}$$

$$p_F = \frac{\left(64\eta^2 + 204\eta + 17\right) t}{9(46\eta + 11)}.$$

These prices are inconsistent with $p_E \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(88\eta^2 + 453\eta + 200\right) t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_B = \frac{\left(139\eta^2 + 426\eta + 176\right) t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_C = \frac{(31\eta + 64)t}{9(11\eta + 8)}$$

$$p_D = \frac{\left(43\eta^2 + 129\eta + 56\right) t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_E = \frac{2\left(11\eta^2 + 69\eta + 34\right) t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_F = \frac{(22\eta + 73)t}{9(11\eta + 8)}.$$

These prices are inconsistent with $p_A < p_F$, so there is no such equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(233\eta^2 + 408\eta + 100\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_B = \frac{\left(263\eta^2 + 390\eta + 88\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_C = \frac{\left(103\eta^2 + 150\eta + 32\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_D = \frac{2\left(31\eta^2 + 69\eta + 14\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_E = \frac{\left(29\eta^2 + 165\eta + 34\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_F = \frac{\left(58\eta^2 + 177\eta + 50\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}.$$

Comparing, these prices are consistent with $p_E < p_F < p_A$, so this case is a candidate for an equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equation 6, then taking first-order conditions and solving yields:

$$p_A = \frac{(44\eta + 203)t}{342}$$

$$p_B = \frac{(32\eta t + 215t)}{342}$$

$$p_C = \frac{5(2\eta + 17)t}{171}$$

$$p_D = \frac{4(2\eta + 17)t}{171}$$

$$p_E = \frac{(11\eta + 65)t}{171}$$

$$p_F = \frac{(28\eta + 67)t}{171}.$$

These prices are inconsistent with $p_E \geq p_F$, so there is no such equilibrium.

We have only one candidate for equilibrium (case (iii)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equation 6. In this profile, prices are ordered as follows:

$$p_B > p_A > p_C > p_F > p_D > p_E.$$

Hence, the worst-off population member is the one who is just indifferent between A and B. This person's position, x^*_{AB} is characterized by:

$$p_A + x^*_{AB}t = p_B + \left(\frac{1}{6} - x^*_{AB}\right)t$$

or

$$x^*_{AB} = \frac{19\eta^2 + 30\eta + 8}{348\eta^2 + 288\eta + 48}.$$

Plugging this in, the person indifferent between A and B prefers to purchase the good if:

$$1 - p_B - x^*_{AB}t \geq 0$$

which is true if and only if:

$$\frac{\eta^2(1044 - 583t) + \eta(864 - 870t) - 200t + 144}{36\left(29\eta^2 + 24\eta + 4\right)} \geq 0$$

or

$$\eta^2(1044 - 583t) + \eta(864 - 870t) - 200t + 144 \geq 0.$$

The LHS is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$ the inequality holds if and only if:

$$461\eta^2 - 6\eta - 56 \geq 0,$$

which is true for any $\eta \in [1, 2]$.

The equilibrium rents are

$$v^{\mathbf{ABCF},DE}(\eta) = \frac{(2408 + 26576\eta + 100262\eta^2 + 146966\eta^3 + 71201\eta^4 + 6728\eta^5)t}{324(4 + 24\eta + 29\eta^2)^2}$$

$$v^{ABCF,\mathbf{DE}}(\eta) = \frac{(820 + 9208\eta + 34069\eta^2 + 44527\eta^3 + 14503\eta^4 + 841\eta^5)t}{162(4 + 24\eta + 29\eta^2)^2}.$$

The incremental returns are:

$$\mathrm{IR}_{\text{att}}^{\text{pop}}(\eta) = v^{\mathbf{ABCF},DE}(\eta) - v^{\mathbf{ABC},DEF}(\eta)$$
$$= \frac{\left(1291776\eta^9 + 10238420\eta^8 + 22459372\eta^7 + 23035725\eta^6 + 13031928\eta^5 + 4314594\eta^4 + 833112\eta^3 + 86872\eta^2 + 3776\eta\right) Nt}{1296\left(8\eta^2 + 6\eta + 1\right)^2 \left(29\eta^2 + 24\eta + 4\right)^2}$$

and

$$\mathrm{IR}_{\text{def}}^{\text{pop}}(\eta) = v^{ABC,\mathbf{DEF}}(\eta) - v^{ABCF,\mathbf{DE}}(\eta)$$
$$= \frac{\left(1291776\eta^9 + 8999020\eta^8 + 17389508\eta^7 + 16381611\eta^6 + 8805816\eta^5 + 2828202\eta^4 + 535560\eta^3 + 55064\eta^2 + 2368\eta\right) Nt}{1296\left(8\eta^2 + 6\eta + 1\right)^2 \left(29\eta^2 + 24\eta + 4\right)^2}.$$

Proof of Proposition 6.2.

(i) Differentiating the rents, we have

$$\frac{\partial v^{\mathbf{ABC},DEF}(\eta)}{\partial \eta} = \frac{\left(2048\eta^6 + 4608\eta^5 - 57608\eta^4 - 66596\eta^3 - 28434\eta^2 - 5485\eta - 408\right) Nt}{648\left(8\eta^2 + 6\eta + 1\right)^3}$$

This is negative if the numerator is negative. To see that this is the case, notice that for $\eta \in [1, 2]$, $2048\eta^6 < 57608\eta^4$ and $4608\eta^5 < 66596\eta^3$, so the positive terms are more than off-set by the negative terms.

$$\frac{\partial v^{ABC,\mathbf{DEF}}(\eta)}{\partial \eta} = \frac{\left(8192\eta^6 + 18432\eta^5 - 19400\eta^4 - 26228\eta^3 - 9786\eta^2 - 1501\eta - 84\right) Nt}{648\left(8\eta^2 + 6\eta + 1\right)^3}$$

To see that it is decreasing and then increasing, note that at $\eta = 1$ this derivative is $-\frac{Nt}{72} < 0$ and at $\eta = 2$ it is $\frac{91943Nt}{9841500} > 0$. To see that it is convex, differentiate again to get:

$$\frac{\partial^2 v^{ABC,\mathbf{DEF}}(\eta)}{\partial \eta^2} = \frac{\left(580736\eta^5 + 605232\eta^4 + 235552\eta^3 + 40072\eta^2 + 2472\eta + 11\right) t}{648\left(8\eta^2 + 6\eta + 1\right)^4},$$

which is clearly positive for $\eta \in [1, 2]$.

$$\frac{\partial v^{\mathbf{ABCF},DE}(\eta)}{\partial \eta} = \frac{\left(97556\eta^6 + 242208\eta^5 - 354903\eta^4 - 574398\eta^3 - 274260\eta^2 - 57528\eta - 4640\right) Nt}{162\left(29\eta^2 + 24\eta + 4\right)^3}$$

To see that it is decreasing and then increasing, note that at $\eta = 1$ this derivative is $-\frac{5Nt}{162} < 0$ and at $\eta = 2$ it is $\frac{23Nt}{7056} > 0$. To see that it is convex, differentiate again to get:

$$\frac{\partial^2 v^{\mathbf{ABCF},DE}(\eta)}{\partial \eta^2} = \frac{\left(1919539\eta^5 + 2572173\eta^4 + 1451984\eta^3 + 446168\eta^2 + 76368\eta + 5776\right) t}{9\left(29\eta^2 + 24\eta + 4\right)^4},$$

which is clearly positive for $\eta \in [1, 2]$.

$$\frac{\partial v^{ABCF,\mathbf{DE}}(\eta)}{\partial \eta} = \frac{\left(24389\eta^6 + 60552\eta^5 - 578319\eta^4 - 675306\eta^3 - 266772\eta^2 - 43560\eta - 2528\right) Nt}{162\left(29\eta^2 + 24\eta + 4\right)^3}$$

To see that this is negative, notice that for any $\eta \in [1, 2]$, $24389\eta^6 < 578319\eta^4$ and $60552\eta^5 < 675306\eta^3$, so the positive terms are more than off-set by the negative terms.

(ii) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{att}}(\eta)}{\partial \eta} = \frac{Nt}{648\left(8\eta^2 + 6\eta + 1\right)^3 \left(29\eta^2 + 24\eta + 4\right)^3}$$

$$\Big[149846016\eta^{12} + 709185024\eta^{11} + 1804009768\eta^{10} + 3180547444\eta^9 + 3868730394\eta^8 + 3194448545\eta^7$$

$$+ 1799287064\eta^6 + 696886116\eta^5 + 185369396\eta^4 + 33247640\eta^3 + 3837552\eta^2 + 256864\eta + 7552\Big]$$

and

$$\frac{\partial \mathrm{IR}_{\text{def}}^{\text{pop}}(\tau)}{\partial \tau} = \frac{Nt}{648\left(8\eta^2+6\eta+1\right)^3\left(29\eta^2+24\eta+4\right)^3}$$

$$\Big[149846016\eta^{12}+709185024\eta^{11}+1938493592\eta^{10}+3288362780\eta^{9}+3601960590\eta^{8}+2665205479\eta^{7}$$

$$+1372937176\eta^{6}+498251100\eta^{5}+126687292\eta^{4}+22024216\eta^{3}+2485200\eta^{2}+163424\eta+4736\Big],$$

both of which are clearly positive.

(iii) In the event that an interior territory is vulnerable, violence is zero. Hence, it suffices to focus on the case of a border territory being vulnerable.

First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, we have:

$$\mathrm{IR}_{\text{att}}^{\text{pop}}(\eta)-\mathrm{IR}_{\text{def}}^{\text{pop}}(\eta) = \frac{\left(619700\eta^8+2534932\eta^7+3327057\eta^6+2113056\eta^5+743196\eta^4+148776\eta^3+15904\eta^2+704\eta\right)Nt}{648\left(8\eta^2+6\eta+1\right)^2\left(29\eta^2+24\eta+4\right)^2},$$

which is clearly positive.

Thus, expected violence is

$$\frac{\mathrm{IR}_{\text{def}}^{\text{pop}}(\eta)^2}{\mathrm{IR}_{\text{at}}^{\text{pop}}(\eta)} = \frac{\left(1291776\eta^9+8999020\eta^8+17389508\eta^7+16381611\eta^6+8805816\eta^5+2828202\eta^4+535560\eta^3+55064\eta^2+2368\eta\right)^2 Nt}{1296(8\eta^2+6\eta+1)^2(29\eta^2+24\eta+4)^2(1291776\eta^8+10238420\eta^7+22459372\eta^6+23035725\eta^5+13031928\eta^4+4314594\eta^3+833112\eta^2+86872\eta+3776)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial \eta}\frac{\mathrm{IR}_{\text{def}}^{\text{trans}}(\eta)^2}{\mathrm{IR}_{\text{at}}^{\text{pop}}(\eta)} = \frac{\left(1291776\eta^8+8999020\eta^7+17389508\eta^6+16381611\eta^5+8805816\eta^4+2828202\eta^3+535560\eta^2+55064\eta+2368\right)Nt}{648(8\eta^2+6\eta+1)^3(29\eta^2+24\eta+4)^3(1291776\eta^8+10238420\eta^7+22459372\eta^6+23035725\eta^5+13031928\eta^4+4314594\eta^3+833112\eta^2+86872\eta+3776)^2}$$

$$\times\Big[193567487164416\eta^{20}+2636013792927744\eta^{19}+14942866864822272\eta^{18}+51819230507149024\eta^{17}+$$
$$122367280695000336\eta^{16}+206967166643804864\eta^{15}+259890474116763824\eta^{14}+249193190122341378\eta^{13}+$$
$$186403803800274835\eta^{12}+110473424142844948\eta^{11}+52399600963659870\eta^{10}+19995986823887684\eta^{9}+$$
$$6143372086092296\eta^{8}+1513744764870168\eta^{7}+296519405242384\eta^{6}+45491642345344\eta^{5}+$$
$$5339965611648\eta^{4}+462318253184\eta^{3}+27777575168\eta^{2}+1032932352\eta+17883136\Big],$$

which is clearly positive.

■

C Factionalization

Proof of Lemma 7.1. First note that

$$v^{\mathbf{2},1,1,1,1} = \frac{145t}{2166} > \frac{t}{36} = v^{1,1,1,1,1,}.$$

Hence, $\text{IR}_{\text{att}}^{1,1,1,1,1,1}$ is minimized at $\pi = 0$. Now note, at $\pi = 0$, we have

$$\text{IR}_{\text{att}}^{1,1,1,1,1,1}(\pi = 0) = \frac{65t}{2166} > \frac{t}{36} = \text{IR}_{\text{def}}^{1,1,1,1}.$$

■

Proof of Proposition 7.3. It follows from Proposition 7.1 that the probability of violence with six factions is 1.

With three factions, there are no safe territories. Thus, from Proposition 7.2, the probability of violence (conditional on a border territory being vulnerable and overall) is

$$\frac{\text{IR}_{\text{def}}^{2,2,2}}{\text{IR}_{\text{att}}^{2,2,2}} = \frac{112,288}{153,579} < 1.$$

Conditional on a border region being vulnerable, the probability of violence is

$$\frac{\text{IR}_{\text{def}}^{3,3}}{\text{IR}_{\text{att}}^{3,3}} = \frac{77}{103}.$$

As border region is vulnerable 2/3 of the time. In the other 1/3 of cases, the probability of violence is zero. Hence, the overall probability of violence is

$$\frac{2}{3} \cdot \frac{77}{103} = \frac{154}{309} < \frac{112,288}{153,579}.$$

■

Proof of Proposition 7.5. Recall from Equation 7 that an arbitrary random variable whose distribution places mass α on zero and mass $1 - \alpha$ on a draw from a symmetric triangular distribution on $[0, b]$ has variance:

$$\frac{(1 + 5\alpha - 6\alpha^2)b^2}{24}.$$

In all of these cases, violence is such a variable. Direct calculation now yields

$$\sigma^2_{1,1,1,1,1,1} = \frac{4225N^2t^2}{28,149,336} \approx 0.00015N^2t^2.$$

$$\sigma^2_{2,2,2} = \frac{5,918,682,193,315,302,400N^2t^2}{10,304,474,604,431,881,718,601} \approx 0.00057N^2t^2,$$

and

$$\sigma^2_{3,3} = \frac{188,548,129N^2t^2}{160,371,415,296} \approx 0.00118N^2t^2.$$

■

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